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Instructions: All solutions should be prepared carefully and neatly. All solution sets shall be completed on this packet and submitted by uploading a scan or picture of your written work to D2L by 11:59 PM on the due date below. **Submit only a single pdf file of your entire packet. Desmos graphs can be submitted separately.** The mobile app called *Genius Scan* works well. Use a PENCIL and if you make a mistake, use an eraser. This assignment is graded on effort, completeness, and neatness for a total of 5 points. Careless presentation (e.g. bad handwriting, pen scribbles, doodles, wasted space, etc) will result in a deduction of points at my discretion. Submitted work that does not demonstrate clearly the process by which one arrived at the answer may result in a loss of points. Any parts to any questions that are not answered will also result in a loss of points. Academic dishonesty will not be tolerated.

Problem Set I

MAT 181 – Calculus I

Due: Friday, February 2 by 11:59 PM on D2L

Read: Sections 1.1–1.4

Solutions!

1. (This question spans two pages.) Answer the following prerequisite questions:

(a) Evaluate the following expressions exactly, no decimal approximations:

$2(-3)^2 =$ $2(9)=18$

$-2\sqrt[3]{125} =$ $(-2)(5)=-10$

$\sqrt{64\cdot 8\cdot 2} =$ $\sqrt{64\cdot 16}=8\cdot 4=32$

$\sin\left(\frac{\pi}{3}\right) =$ $\sqrt{3}/2$

$\log_2(32) =$ 5

$3\cdot 8^{2/3} =$ $3(8^{1/3})^2=3(2)^2=12$

$\tan\left(\frac{3\pi}{4}\right) =$ -1

$\arcsin\left(-\frac{1}{2}\right) =$ $-\pi/6$

$\sin\left(\cos^{-1}(1/2)\right) =$ $\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

$\ln(1) =$ 0

$\sec\left(-\frac{5\pi}{3}\right) =$ 2

$\log_{10}\left(\frac{1}{10000}\right) =$ -5

$\log_{19}(1) =$ 0

(b) Simplify the following expression exactly so that there are no negative exponents and no radical forms:

$$\frac{3\sqrt[4]{y^5}\sqrt{z}(x^{-2})^6}{3^{-2}yx^{-3}\sqrt[3]{z}} = \frac{3\cdot y^{5/4}\cdot z^{1/2}\cdot x^{-12}}{3^{-2}\cdot y\cdot x^{-3}\cdot z^{1/3}} = \frac{3\cdot 3^2\cdot z^{1/2}z^{-1/3}y^{5/4}y^{-1}}{x^{-3}x^{12}} = \boxed{\frac{27z^{1/6}y^{1/4}}{x^9}}$$

(c) Simplify the following expression exactly (no decimal approximations):

$$\frac{\left(\sqrt{16}+2\right)^2}{\sqrt{25-4^2}} = \frac{(4+2)^2}{\sqrt{25-16}} = \frac{6^2}{\sqrt{9}} = \frac{36}{3} = \boxed{12}$$

(d) Simplify the following expression exactly (no decimal approximations):

$$2\cdot 5^{-1}+3^{-1} = 2\cdot\frac{1}{5}+\frac{1}{3} = \frac{2}{5}+\frac{1}{3} = \frac{6}{15}+\frac{5}{15} = \boxed{\frac{11}{15}}$$

(e) Let $x \neq 0$ and $y \neq 0$. Simplify the following expression completely:

$$\frac{x^{-1}y^{-1}}{x^{-1}+y^{-1}} = \frac{\frac{1}{x}\cdot\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}} = \frac{\frac{1}{xy}}{\frac{y+x}{xy}} = \boxed{\frac{1}{y+x}}$$

(f) Let $f(x) = 2x^2+3x-2$. Is this function algebraic or transcendental? What are the zeros of this function? On what interval(s) is this function positive? Use interval notation and show all algebraic work.

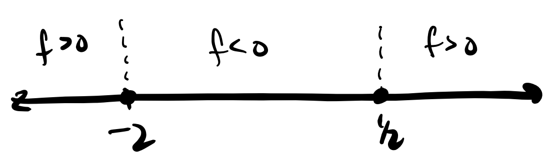
· Zeros: $2x^2+3x-2=0$

$\Rightarrow (2x-1)(x+2)=0$

$\Rightarrow 2x-1=0 \quad x+2=0$

$\Rightarrow \boxed{x=\frac{1}{2},\ x=-2}$

Check Sign:



· $f(-3)=2(-3)^2+3(-3)-2=18-9-2=7>0$

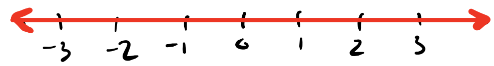
· $f(0)=2(0)^2+3(0)-2=-2<0$

· $f(1)=2(1)^2+3(1)-2=3>0$

$\boxed{f(x) \text{ is positive on } (-\infty,-2)\cup(1/2,\infty)}$

2. For each of the following functions, determine if it is a polynomial, rational, algebraic, or transcendental function. Then, find the domain of each function. Write your answers as a portion of the number line and in interval notation:

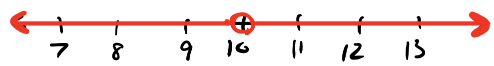
transcendental (a) $f(x) = x^3 + 3^x$



The domains of x^3 and 3^x are all real #'s.

$\to$ Domain: $(-\infty, \infty)$

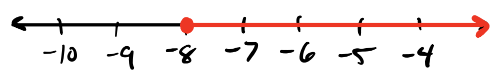
rational (b) $f(x) = \dfrac{1}{x-10}$



$x - 10 \neq 0$

$\Rightarrow x \neq 10 \to$ Domain: $(-\infty, 10) \cup (10, \infty)$

algebraic (c) $f(x) = \sqrt{x+8}$



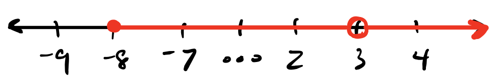
$x + 8 \geq 0$

$\Rightarrow x \geq -8 \to$ Domain: $[-8, \infty)$

algebraic (d) $f(x) = \dfrac{\sqrt{x+8}}{x-3}$

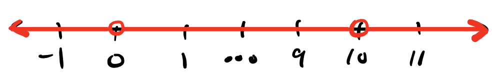
Numerator: $x + 8 \geq 0$ $\Rightarrow x \geq -8$

Denominator: $x - 3 \neq 0$ $\Rightarrow x \neq 3$



$\to$ Domain: $[-8, 3) \cup (3, \infty)$

rational (e) $f(x) = \dfrac{x+4}{x^2 - 10x}$



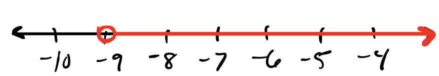
$x^2 - 10x \neq 0$

$x(x-10) \neq 0$

$x \neq 0,\ x \neq 10 \to$ Domain: $(-\infty, 0) \cup (0, 10) \cup (10, \infty)$

transcendental (f) $f(x) = \log_2(x+9)$

• We must have $x + 9 > 0 \Rightarrow x > -9$

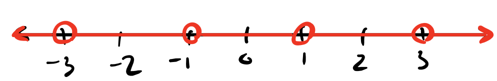


$\to$ Domain: $\boxed{(-9, \infty)}$

transcendental (g) $f(x) = \tan\left(\frac{\pi}{2}x\right)$

• f has vertical asymptotes at $\ldots -5, -3, -1, 1, 3, 5, \ldots$

i.e. every odd integer



$\to$ Domain: $\boxed{\ldots(-5,-3) \cup (-3,-1) \cup (-1,1) \cup (1,3) \cup (3,5)\ldots \text{ or } \mathbb{R} \setminus \mathbb{Z}_{odd}}$

3. (This problem spans two pages.) Let $f(x) = 2x^2 - 4x - 6$ and $g(x) = \sqrt{x+6}$. Using these two functions, answer the questions below.

   (a) What are the domains of each of these functions? Submit your answer in interval notation.

Domain of f: $(-\infty, \infty)$ since it is a quadratic function defined for all x.

Domain of g: $[-6, \infty)$ As long as $x+6 \geq 0 \Rightarrow x \geq -6$, the function will output a real number.

   (b) Find an expression for $f(x-2)$. Simplify as much as possible.

$$f(x-2) = 2(x-2)^2 - 4(x-2) - 6$$
$$= 2(x^2 - 4x + 4) - 4x + 8 - 6$$
$$= 2x^2 - 8x + 8 - 4x + 8 - 6$$
$$= \boxed{2x^2 - 12x + 10}$$

   (c) Find an expression for $g(x^2 + 30)$. Simplify as much as possible.

$$g(x^2+30) = \sqrt{(x^2+30)+6}$$
$$= \boxed{\sqrt{x^2+36}}$$

Can't reduce any further.

   (d) Find the zeros of $f(x)$.

Set $f(x) = 0$ and solve for x:

$$2x^2 - 4x - 6 = 0$$
$$\Rightarrow 2(x^2 - 2x - 3) = 0$$
$$\Rightarrow 2(x-3)(x+1) = 0$$

$\Rightarrow x - 3 = 0$ $\quad x + 1 = 0$

$x = 3$ $\quad x = -1$

Zeros: $\boxed{x = -1,\ x = 3}$

(e) Find the inverse function of $g(x)$ and determine the domain and range of the inverse function.

$g(x) = \sqrt{x+6}$ Domain: $[-6, \infty)$ Range: $[0, \infty)$

$y = \sqrt{x+6}$

$x = \sqrt{y+6}$

$x^2 = y+6$

$y = x^2 - 6$

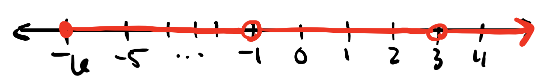
$\Rightarrow \boxed{g^{-1}(x) = x^2 - 6}$ Domain: $[0, \infty)$ Range: $[-6, \infty)$

(f) Find an expression for $(g/f)(x)$. Find the domain of this function.

$$(g/f)(x) = \frac{\sqrt{x+6}}{2x^2-4x-6}$$

Numerator: must have $x \geq -6$ to maintain a real #.

Denominator: must have $x \neq -1$, $x \neq 3$ so as to not divide by zero.



Domain of (g/f):

$[-6, -1) \cup (-1, 3) \cup (3, \infty)$

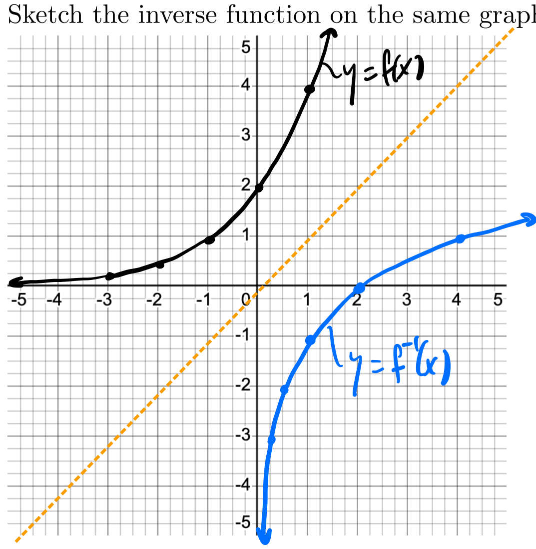
(g) Find an expression for the function $(f \circ g)(x)$ and $(g \circ f)(x)$. Simplify both expressions as much as possible.

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x+6}) = 2(\sqrt{x+6})^2 - 4(\sqrt{x+6}) - 6$$
$$= 2(x+6) - 4\sqrt{x+6} - 6$$
$$= 2x + 12 - 4\sqrt{x+6} - 6$$
$$= \boxed{2x - 4\sqrt{x+6} + 6}$$

$$(g \circ f)(x) = g(f(x)) = g(2x^2 - 4x - 6) = \sqrt{2x^2 - 4x - 6 + 6}$$
$$= \boxed{\sqrt{2x^2 - 4x}}$$

4. Consider the following questions about graphing functions:

   (a) Write down a **transcendental** function that has a y-intercept at $(0, 2)$, is always increasing, and has a horizontal asymptote at $y = 0$. (There is more than one answer.) Sketch this function on the graph below. What is the domain and range of this function? What is the inverse of this function? Sketch the inverse function on the same graph.



Answers vary but the most straightforward solution is $f(x) = 2^{x+1}$.

Domain: $(-\infty, \infty)$ Range: $(0, \infty)$

Inverse: $y = 2^{x+1}$

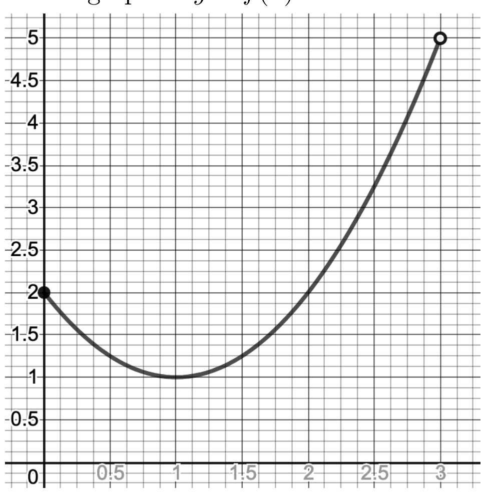
$\Rightarrow x = 2^{y+1}$

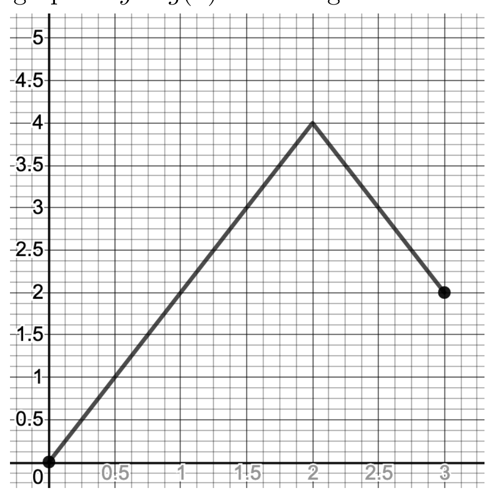
$\Rightarrow \log_2(x) = y+1$

$\Rightarrow y = \log_2(x) - 1$

$\boxed{f^{-1}(x) = \log_2(x) - 1}$

   (b) Consider the graph of $y = f(x)$ on the left and the graph of $y = g(x)$ on the right.





Answer the following questions. If the value does not exist, write DNE.

   (a) What is the domain and range of $f(x)$? Domain: $[0,3)$ Range: $[1,5)$

   (b) On what interval(s) is $f(x)$ decreasing? $(0,1)$

   (c) What is slope of the line connecting the points $(0, g(0))$ and $(1, g(1))$? $m = 2$

   (d) Is the value $x = 0$ in the domain of $(f/g)(x)$? $\frac{f(0)}{g(0)} = \frac{2}{0} = \text{DNE} \rightarrow$ No

   (e) Compute $(g \circ f)(2)$. $= g(f(2)) = g(2) = 4$

   (f) Compute $(f \circ g)(2)$. $= f(g(2)) = f(4) = \text{DNE}$ (not in domain)

5. Application Problem: Suppose you deposit \$1500 into a savings account that yields 3% annual interest and it is compounded quarterly (i.e. every three months). Then the total amount, call it $A(t)$, in your bank account t years after the initial deposit is given by the following exponential function

$$A(t) = 1500\left(1 + \frac{0.03}{4}\right)^{4t}$$

(a) How much money would you have after six years? Round your answer to two decimal places.

Plug $t=6$ into the function

$$A(6) = 1500\left(1+\frac{0.03}{4}\right)^{4\cdot 6} = 1500(1.0075)^{24} \approx \boxed{\$1794.62}$$

(b) How long many years would it take for your money to triple? Round your answer to two decimal places. I'm asking for an exact answer to within two decimal places, *not a "ball-park" estimate.* Show all relevant work.

Plug 4500 in for $A(t)$ and solve for t:

$$4500 = 1500\left(1+\frac{.03}{4}\right)^{4t}$$

$$\Rightarrow \quad 3 = (1.0075)^{4t}$$

$$\Rightarrow \quad \ln(3) = \ln(1.0075)^{4t}$$

$$\Rightarrow \quad \ln(3) = 4\cdot t \ln(1.0075)$$

$$\frac{\ln(3)}{\ln(1.0075)} = 4\cdot t$$

$$\Rightarrow t = \frac{\ln(3)}{4\cdot\ln(1.0075)}$$

$$\Rightarrow \boxed{t = 36.758 \text{ years}}$$

(c) Plot this function in Desmos for $x \in [-5, 50]$ and $y \in [-1000, 6000]$. Upload this picture to D2L when you submit the assignment. Judging from the graph, is this function monotonic? Justify your answer.

Yes, it is an exponential function with base $b = 1.0075$.