EXAM III REVIEW PROBLEMS (SECTIONS 3.5, 4.1–4.4, AND 4.6)

Disclaimer: I will *not* provide detailed solutions to these problems. Also, this worksheet is meant to be a helpful sample problem set and is *not* supposed to be a comprehensive review of all possible material that could be on the test. Study the lecture notes and worksheets to get a full review of the test material.

1.] §3.5: For the implicitly defined curves below, find the slope $\frac{dy}{dx}$. If a point is given, find the slope of the tangent line at the point.

a.) $\sin(y) = 5x^4 - 5$ at $(1, \pi)$

b.) $x^3 = \dfrac{x+y}{x-y}$

c.) $\sin(y) + 5x = y^2$ at $\left(\dfrac{\pi^2}{5}, \pi\right)$

d.) $\cos(x-y) + \sin(y) = \sqrt{2}$ at $\left(\dfrac{\pi}{2}, \dfrac{\pi}{4}\right)$.

2.] §3.5: For the implicitly defined curves below, find the second derivative $\frac{d^2y}{dx^2}$.

a.) $x + y^2 = 1$

b.) $e^{2y} + x = y$

3.] §4.1: Sketch the graph of a continuous function f on $[0,4]$ such that $f'(x) = 0$ for $x = 1$ and $x = 2$; f has an absolute maximum at $x = 4$; f has an absolute minimum at $x = 0$; and f has a local minimum at $x = 2$.

4.] §4.1: Find the critical points of the following functions on the domain or on the interval provided.

a.) $f(x) = 3x^2 - 4x + 2$

b.) $f(x) = \dfrac{x}{x^2+1}$

c.) $f(x) = x^2\sqrt{x+1}$ on $[-1, 1]$

5.] §4.1: Find the critical points of f on the given interval and determine the absolute extrema on the interval using the Closed Interval Method.

a.) $f(x) = x^2 - 10$ on $[-2, 3]$

b.) $f(x) = \sin(3x)$ on $[-\pi/4, \pi/3]$

c.) $f(x) = \dfrac{4x^3}{3} + 5x^2 - 6x$ on $[-4, 1]$

6.] §4.2: Determine if the Rolle's Theorem applies to the following functions on the given interval. If it does, find the approximate point(s) that are guaranteed to exist by the theorem.

a.) $f(x) = x(x-1)^2$ on $[0, 1]$

b.) $f(x) = 1 - x^{2/3}$ on $[-1, 1]$

c.) $f(x) = x^3 - x^2 - 5x - 3$ on $[-1, 3]$

7.] §4.2: Determine if the Mean Value Theorem applies to the following functions on the given interval. If it does, find the approximate point(s) that are guaranteed to exist by the theorem.

a.) $f(x) = 7 - x^2$ on $[-1, 2]$

b.) $f(x) = e^x$ on $[0, \ln(4)]$

c.) $f(x) = 2x^{1/3}$ on $[-8, 8]$

8.] §4.3: Find the intervals on which the following functions are increasing and decreasing:

a.) $f(x) = 3\cos(3x)$ on $[-\pi, \pi]$

b.) $f(x) = -12x^5 + 75x^4 - 80x^3$

c.) $f(x) = \arctan(x^2)$

9.] §4.3: Locate the critical points of f and use the first derivative test to determine the location of local max and min values. Then, on the interval provided, find the absolute extrema using the Closed Interval Method.

a.) $f(x) = x^2 + 3$ on $[-3, 2]$

b.) $f(x) = x\sqrt{9 - x^2}$ on $[-3, 3]$

c.) $f(x) = \sqrt{x}\ln(x)$ on $(0, \infty)$

10.] §4.3: Find the intervals on which the following functions are concave up and concave down:

a.) $f(x) = x^4 - 2x^3 + 1$

b.) $f(x) = e^x(x - 3)$

c.) $f(x) = 3x^5 - 30x^4 + 80x^3 + 100$

11.] §4.3: Locate the critical points of f and use the second derivative test to determine the location of local max and min values.

a.) $f(x) = x^3 - 3x^2$

b.) $f(x) = e^x(x - 7)$

c.) $f(x) = 2x^2\ln(x) - 11x^2$

12.] §4.3: Consider the function $f(x) = x^3 - 6x^2 + 9x$. Identify all local extrema, inflection points, and x and y intercepts.

13.] §4.3: Consider the function $f(x) = (x - 6)(x + 6)^2$. Identify all local extrema, inflection points, and x and y intercepts.

14.] §4.4: Evaluate the following limits:

a.) $\displaystyle\lim_{x\to 2} \frac{x^2 - 2x}{8 - 6x + x^2}$

b.) $\displaystyle\lim_{x\to 0} \frac{3\sin(4x)}{5x}$

c.) $\displaystyle\lim_{x\to \infty} \frac{5}{x^{3/2}}$

d.) $\displaystyle\lim_{x\to 0} \frac{1 - \cos(3x)}{8x^2}$

e.) $\displaystyle\lim_{x\to \infty} \frac{e^{1/x} - 1}{1/x}$

15.] §4.6: Find the positive numbers x and y satisfying the equation $xy = 12$ such that the sum $2x + y$ is as small as possible.

16.] §4.6: A farmer has 120 feet of fencing to construct a rectangular pen up against the straight side of a barn, using the barn for one side of the pen. The length of the barn is 100 feet. Determine the dimensions of the rectangle of maximum area that can be enclosed under these conditions.