By signing below, you attest that you have neither given nor received help of any kind on this exam.

Signature: Brooks Emerick Printed Name: Brooks Emerick

Instructions: Show work to get full credit (the correct answer may NOT be enough). Do all your work on the paper provided. Write clearly! Double check your answers!

You will **not** receive full credit for using methods other than those discussed in class.

Calculators are not permitted.

Exam III

MAT 181 – Calculus I

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solutions!

Problem Number	Available Points	Your Points
1	6	6
2	8	8
3	7	7
4	6	6
5	6	4
Total	33	33

1. For the following questions, let $f(x) = \frac{1}{2}x^3 - 2x^2$. [(6)]

(a) Find the values on the interval $[-2, 2]$ on which the function f attains its absolute minimum value and its absolute maximum value. [3]

$f'(x) = \frac{3}{2}x^2 - 4x = 0$

$x\left(\frac{3}{2}x - 4\right) = 0$

$\boxed{x = 0}$ $\frac{3}{2}x - 4 = 0$

$\frac{3}{2}x = 4$

$\boxed{x = \frac{8}{3}}$ outside interval

Critical pts: $c = 0$

Closed interval method:

$f(-2) = \frac{1}{2}(-2)^3 - 2(-2)^2 = -4 - 8 = -12$

$f(0) = \frac{1}{2}(0)^3 - 2(0)^2 = 0$

$f(2) = \frac{1}{2}(2)^3 - 2(2)^2 = 4 - 8 = -4$

Abs. Max at $x = 0$
Abs min at $x = -2$

(b) Does f satisfy the hypotheses of Rolle's Theorem on the interval $[-2, 2]$? If yes, justify each part of the theorem. If no, explicitly state which part(s) of the hypothesis is not met. If it does satisfy the hypothesis, determine the value c that is guaranteed to exist. [3]

✓ 1.) $f(x) = \frac{1}{2}x^3 - 2x^2$ is a polynomial so it is cont. on $[-2,2]$

✓ 2) $f'(x) = \frac{3}{2}x^2 - 2x$ is a polynomial so $f(x)$ is differentiable on $(-2,2)$

✗ 3.) $f(-2) = -12$
$f(2) = -4$
Not Equal

f does not satisfy the hypothesis of Rolles' Thm.

2. Suppose that $f(x)$ is a continuous function everywhere, and the **FIRST DERIVATIVE** of $f(x)$ is given by [(8)]

$$f'(x) = \frac{x^2 - 3x - 4}{x^{1/3}}$$

(a) Find all the critical points of f. [3]

$f'(x) = 0$:

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = -1, \ x = 4$

$f'(x) = \text{DNE}$:

$x^{1/3} = 0$

$x = 0$

critical pts:

$\boxed{c_1 = -1, \ c_2 = 0, \ c_3 = 4}$

(b) Conduct a sign analysis on the number line below to find the intervals of monotonicity of $f(x)$. Your final answers should be in interval notation. [3]

$f'(x) = \frac{(x+1)(x-4)}{x^{1/3}}$

$f'(-2) = \frac{(-)(-)}{(-)} < 0$

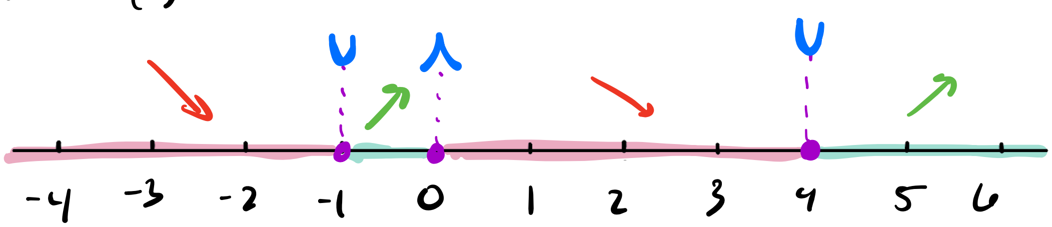
$f'(-\frac{1}{2}) = \frac{(+)(-)}{(-)} > 0$

$f'(1) = \frac{(+)(-)}{(+)} < 0$

$f'(5) = \frac{(+)(+)}{(+)} > 0$

Summary

- Inc on $(-1,0) \cup (4,\infty)$
- Dec on $(-\infty,-1) \cup (0,4)$



(c) **Using the first derivative test**, determine the x-values that correspond to local extrema of $f(x)$, and indicate whether the point corresponds to a local min or local max. [2]

- Local max at $x = 0$
- Local min at $x = -1$ and $x = 4$.

3. Consider the function $f(x) = x^3 - \frac{3}{2}x^2 - 36x$. [(7)]

(a) Find all **critical points** and **candidate inflection points** of this function. [3]

Critical Pts:

$f'(x) = 3x^2 - 3x - 36 = 0$

$3(x^2 - x - 12) = 0$

$3(x-4)(x+3) = 0$

$\boxed{c_1 = -3,\ c_2 = 4}$

Candidate Inflection Pts:

$f''(x) = 6x - 3 = 0$

$x = \frac{3}{6}$

$\boxed{x = \frac{1}{2}}$

(b) Determine the location of all local maxima and minima of this function using the **second derivative test**. [2]

$f''(x) = 6x - 3$

- $f''(-3) = 6(-3) - 3 = -18 - 3 = -21 < 0$ $\hookrightarrow$ $\boxed{\text{Local max at } x = -3}$
- $f''(4) = 6(4) - 3 = 24 - 3 = 21 > 0$ $\hookrightarrow$ $\boxed{\text{local min at } x = 4}$

(c) Determine the intervals of concavity for $f(x)$ by conducting a sign analysis on the number line below. Your final answers should be in interval notation. [2]

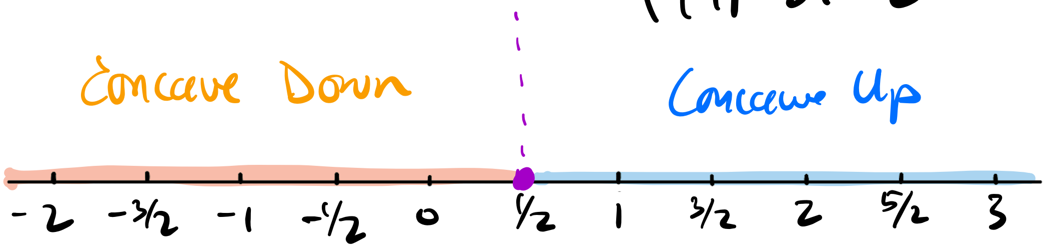
$f''(x) = 6x - 3$

Summary:

- Concave up on $(\frac{1}{2}, \infty)$
- Concave down on $(-\infty, \frac{1}{2})$

$f''(-3) = -21 < 0$

$f''(4) = 21 > 0$



4. Compute the following limits. If you use L'Hôpital's Rule, state the indeterminate form. [(6)]

(a) $\lim_{x\to 0} \frac{3-2^x}{2x+3}$ [2]

$$\lim_{x\to 0} \frac{3-2^x}{2x+3} = \frac{3-2^0}{2(0)+3} = \frac{3-1}{0+3} = \boxed{\frac{2}{3}}$$

(b) $\lim_{x\to 2} \frac{e^{3x-6}-1}{x^2-4}$ [2]

$$\lim_{x\to 2} \frac{e^{3x-6}-1}{x^2-4} \quad \left[= \frac{e^{6-6}-1}{2^2-4} = \frac{e^0-1}{4-4} = \frac{0}{0}\right]$$

L'H

$$\lim_{x\to 2} \frac{3e^{3x-6}-0}{2x} = \frac{3e^{3(2)-6}}{2(2)} = \frac{3e^0}{4} = \boxed{\frac{3}{4}}$$

(c) $\lim_{x\to 0^+} x^2 \csc(2x)$ [2]

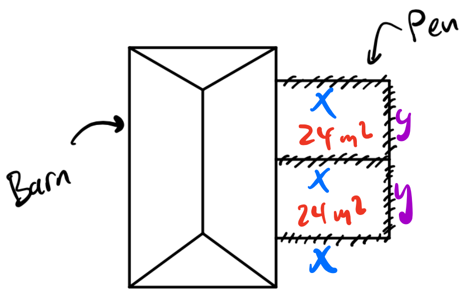
$$\lim_{x\to 0^+} \frac{x^2}{\sin(2x)} \quad \left[= \frac{0}{0}\right]$$

L'H

$$\lim_{x\to 0^+} \frac{2x}{2\cos(2x)} = \frac{2(0)}{2\cos(0)} = \frac{0}{2(1)} = \frac{0}{2} = \boxed{0}$$

5. Farmer Kenny wants to build a rectangular pen against his barn. He decided that he does not need fencing along the barn, but does want the pen to be divided into two equal rectangular sections by one interior fence that runs perpendicular to the barn. Each section should have an area of 24 square meters. What is the minimum amount of fencing required and what are the dimensions of the optimal pen? Be sure to justify your answer using the **second derivative test**. [(6)]

Aerial View



Obj. Fcn: Minimize Fencing

$$F = 3x + 2y$$

Constraint: $xy = 24$

$$y = \frac{24}{x} \longrightarrow F(x) = 3x + 2\left(\frac{24}{x}\right)$$

$$F(x) = 3x + \frac{48}{x}$$

$$F'(x) = 3 - \frac{48}{x^2} = 0$$

$$3 = \frac{48}{x^2}$$

$$3x^2 = 48$$

$$x^2 = 16$$

$$\boxed{x = 4\text{ m}}$$

$$y = \frac{24}{4}$$

$$\boxed{y = 6\text{ m}}$$

$$F''(x) = \frac{96}{x^3} \rightarrow F''(4) = \frac{96}{4^3} > 0$$

Minimum

Dimensions: $x = 4$ m
$y = 6$ m

Min Fencing $= 24$ m