By signing below, you attest that you have neither given nor received help of any kind on this exam.

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Instructions: Show work to get full credit (the correct answer may NOT be enough). Do all your work on the paper provided. Write clearly! Double check your answers!

You will **not** receive full credit for using methods other than those discussed in class.

Calculators are not permitted.

Exam II

MAT 181 – Calculus I

Solutions!

Problem Number	Available Points	Your Points
1	4	4
2	6	6
3	5	5
4	7	7
5	14	14
Total	36	36

1. Determine if the following function is differentiable at $x = 1$: [(4)]

$$f(x) = \begin{cases} 2x^3 + 2 & \text{for } x \leq 1 \\ 6x - 1 & \text{for } x > 1 \end{cases}$$

You must justify your answer using appropriate limits or appropriate theorems.

Check Continuity:

- $f(1) = 2(1)^3 + 2 = 4$
- $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 2x^3 + 2 = 2(1)^3 + 2 = 4$
- $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 6x - 1 = 6(1) - 1 = 5$

Therefore, $f(x)$ is not continuous at $x = 1$ due to the limit not existing. Since $f(x)$ is not continuous, it is not differentiable at $x = 1$.

2. [(6)]

(a) Let $y = f(x)$ be a function and let c be a value in the domain of f. Without using the word "prime," "derivative," or "differentiate," write a single English sentence with appropriate punctuation that interprets what is meant by the symbol $f'(c)$. [1]

Answers vary:

1.) $f'(c)$ is the slope of the tangent line to $y=f(x)$ at the point $(c, f(c))$.

2.) $f'(c)$ is the instantaneous rate of change of $f(x)$ when $x = c$.

(b) Use the **limit definition** to compute the derivative function $f'(x)$ for $f(x) = x^3 - 2x + \pi$. Show all work. [5]

- $f(x) = x^3 - 2x + \pi$
- $f(x+h) = (x+h)^3 - 2(x+h) + \pi$

$$= x^3 + 3x^2h + 3xh^2 + h^3 - 2x - 2h + \pi$$

- $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3 - 2x - 2h + \pi) - (x^3 - 2x + \pi)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-2x} - 2h \cancel{+\pi} \cancel{-x^3} \cancel{+2x} \cancel{-\pi}}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 2h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 - 2)}{\cancel{h}}$$

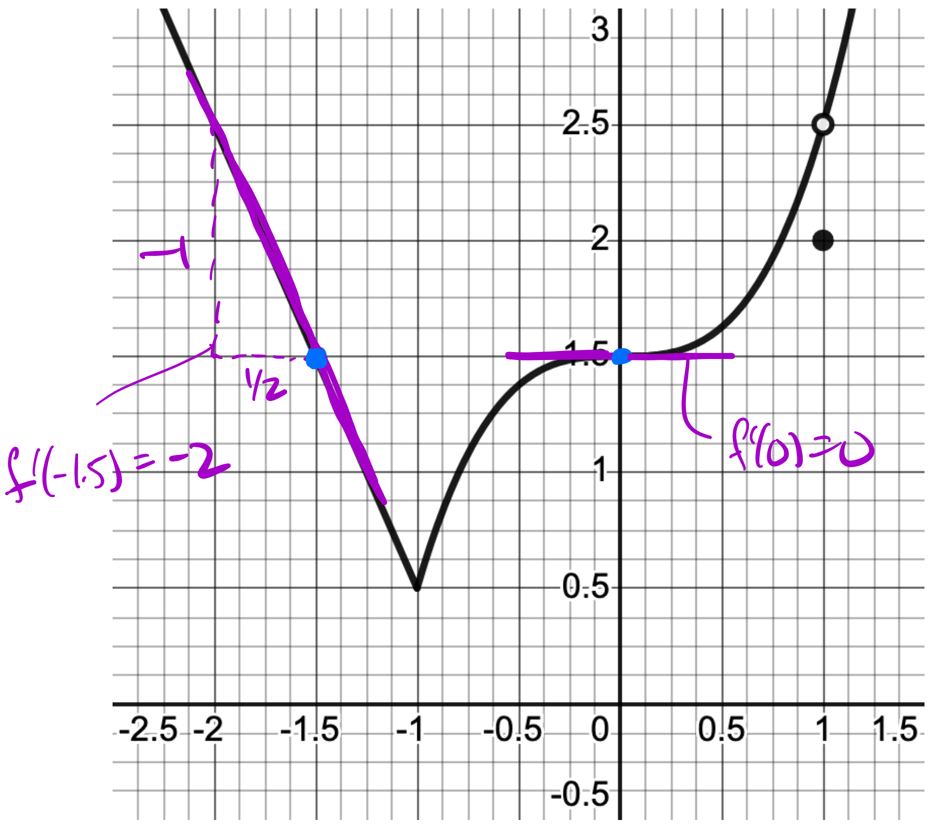
$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 - 2$$

$$= 3x^2 + 3x(0) + 0^2 - 2$$

$$= 3x^2 + 0 + 0 - 2$$

$$\boxed{f'(x) = 3x^2 - 2}$$

3. Let the graph of $y = f(x)$ be given below. [(5)]



(a) Evaluate the following derivatives. If it doesn't exist, write DNE.

i. $f'(-1.5) =$ -2 (same slope as line) [1]

ii. $f'(-1) =$ DNE (cusp; not differentiable) [1]

iii. $f'(0) =$ 0 (slope is horizontal) [1]

iv. $f'(1) =$ DNE (f is not continuous at $x=1$) [1]

(b) Use the graph of f above to evaluate the following limit: [1]

$$\lim_{x \to -1^-} \frac{f(x) - f(-1)}{x+1} = \boxed{-2}$$

Slope on Left-Hand side of $x=-1$

4. Consider the function $f(x) = 12\sqrt{x} - \frac{3}{2}x^2 - x - 2$. [(7)]

(a) On what interval(s) is this function continuous? [1]

The function is an algebraic function that is continuous on its domain. The domain is $[0,\infty)$ due to the square root.

(b) On what interval(s) is this function differentiable? [1]

$f'(x) = 12\left(\frac{1}{2\sqrt{x}}\right) - \frac{3}{2}(2x) - 1 - 0$

$\Rightarrow f'(x) = \frac{6}{\sqrt{x}} - 3x - 1 \rightarrow$ continuous on $(0,\infty)$ $\rightarrow$ Since $f'(x)$ is continuous on $(0,\infty)$, $f(x)$ is differentiable on $(0,\infty)$.

(c) Find the average rate of change of this function over the interval $[0, 4]$. [2]

$[AROC] = \frac{f(4)-f(0)}{4-0}$

$= \frac{-6-(-2)}{4}$

$= \frac{-4}{4}$

$= \boxed{-1}$

$f(4) = 12\sqrt{4} - \frac{3}{2}(4)^2 - 4 - 2$

$= 12(2) - \frac{3}{2}(16) - 4 - 2$

$= 24 - 24 - 6$

$= -6$

$f(0) = 12\sqrt{0} - \frac{3}{2}(0)^2 - 0 - 2$

$= -2$

(d) Find the equation of the tangent line to $f(x)$ at the point $(4, f(4))$. Your final answer should be in slope-intercept form. [3]

Point: $(4, f(4)) = (4, -6)$

Slope: $m = f'(4) = -10$

$f'(x) = \frac{6}{\sqrt{x}} - 3x - 1$

$\Rightarrow f'(4) = \frac{6}{\sqrt{4}} - 3(4) - 1$

$= \frac{6}{2} - 12 - 1$

$= 3 - 12 - 1$

$= -10$

$y = mx + b$

$-6 = -10(4) + b$

$\Rightarrow -6 = -40 + b$

$\Rightarrow 34 = b$

$\Rightarrow$ Eqn of TL: $\boxed{y = -10x + 34}$

5. Find the derivative of the functions below using methods discussed in class. You do not have to simplify. [(14)]

(a) $f(x) = 3^x + 3^3 + x^3$ [2]

$f'(x) = \ln(3)\cdot 3^x + 0 + 3x^2$

(b) $f(x) = e^{4x} + e^4$ [2]

$f'(x) = e^{4x}\cdot 4 + 0$

(c) $f(x) = x^{1/4}\cot(x)$ [2]

$f'(x) = (x^{1/4})'(\cot(x)) + (x^{1/4})(\cot(x))'$

$f'(x) = \frac{1}{4}x^{-3/4}\cot(x) + x^{1/4}(-\csc^2(x))$

(d) $f(x) = \frac{5x}{3} + \sqrt{1-x^2}$ [2]

$f'(x) = \frac{5}{3}(x)' + \frac{1}{2\sqrt{1-x^2}}(1-x^2)'$

$f'(x) = \frac{5}{3} + \frac{1}{2\sqrt{1-x^2}}\cdot(-2x)$

(e) $f(x) = \frac{x^4\sec(x)}{x^2+1}$ [3]

$f'(x) = \frac{(x^4\sec(x))'(x^2+1) - (x^4\sec(x))(x^2+1)'}{(x^2+1)^2}$

$f'(x) = \frac{(4x^3\sec(x) + x^4\sec(x)\tan(x))(x^4+1) - 2x(4x^3\sec(x))}{(x^2+1)^2}$

(f) $f(x) = \cos^3(9x + \sqrt{3x^2-6}) + 21$ [3]

$f'(x) = 3\cos^2(9x+\sqrt{3x^2-6})\cdot(\cos(9x+\sqrt{3x^2-6}))' + 0$

$f'(x) = 3\cos^2(9x+\sqrt{3x^2-6})\cdot(-\sin(9x+\sqrt{3x^2-6}))\cdot(9x+\sqrt{3x^2-6})'$

$f'(x) = 3\cos^2(9x+\sqrt{3x^2-6})\cdot(-\sin(9x+\sqrt{3x^2-6}))\cdot\left(9+\frac{1}{2\sqrt{3x^2-6}}\cdot(6x)\right)$