By signing below, you attest that you have neither given nor received help of any kind on this exam.

Signature: _Brooks Emerick_   Printed Name: _Brooks Emerick_

Instructions: All solutions should be prepared carefully and neatly. All solution sets shall be completed on this packet or on white unlined paper, and handed in at the end of class on the due date below. If you are working remotely, upload pictures of your written work to D2L by 11:59 PM on the due date below. Careless presentation (e.g. bad handwriting, pen scribbles, doodles, wasted space, etc) will result in a deduction of points at my discretion. Use a PENCIL and if you make a mistake, use an eraser. Submitted work that does not demonstrate clearly the process by which one arrived at the answer will not receive credit of any kind. If a problem asks for an Excel solution, then upload your Excel program files to D2L as well. Academic dishonesty will not be tolerated.

TAKE-HOME EXAM I

MAT 362 – OPERATIONS RESEARCH II
DUE: SUNDAY, MARCH 17, 2023 AT 11:59 PM VIA D2L

Solutions!

Problem Number	Available Points	Your Points
1	5	5
2	5	5
3	5	5

1. The Onenote Co. produces a single product at three plants for four customers. The three plants will   [(5)]
produce 60, 80, and 40 units, respectively, during the next time period. The firm has made a commitment
to sell 40 units to customer 1, 60 units to customer 2, and at least 20 units to customer 3. Both customers
3 and 4 also want to buy as many of the remaining units as possible. The net profit associated with
shipping a unit from plant i for sale to customer j is given by the following table:

	Customer 1	Customer 2	Customer 3	Customer 4
Plant 1	$800	$700	$500	$200
Plant 2	$500	$200	$100	$300
Plant 3	$600	$400	$300	$500

Management wishes to know how many units to sell to customers 3 and 4 and how many units to ship
from each of the plants to each of the customers to maximize profits. Formulate this problem as a
transportation problem where the objective function is to be maximized by constructing the appropriate
grid tableau that gives unit profits. Formulate the problem in Excel and use the Solver to find the
solution.

Decision Variables: X_{ij} = # of units shipped from plant i to customer j.

	Customer 1	Customer 2	Customer 3	Customer 4	Supply
Plant 1	0 (800)	60 (700)	0 (500)	0 (200)	60
Plant 2	40 (500)	0 (200)	0 (100)	40 (300)	80
Plant 3	0 (600)	0 (400)	20 (300)	20 (500)	40
Demand	40	60	≥ 20	≥ 0	

Objective Function: Max $Z = \sum_{i=1}^{3} \sum_{j=1}^{4} C_{ij} X_{ij}$. with $X_{ij} \geq 0$.

Excel Solution: $X_{12} = 60$, $X_{21} = 40$, $X_{24} = 40$, $X_{33} = 20$, $X_{34} = 20$

$Z_{opt} = \$90,000$

2. A department head has four subordinates, and four tasks to be performed. The subordinates differ in   [(5)]
   efficiency, and the tasks differ in their intrinsic difficulty. Her estimate of the times each person would
   take to perform each task is given in the table below. How should the tasks be allocated, one to a person,
   ao as to minimize the total time required to complete all of the tasks? What is the minimum total time?
   Solve this problem using the Hungarian method and show all steps.

	Person 1	Person 2	Person 3	Person 4
Task A	18	26	17	11
Task B	13	28	14	26
Task C	10	40	18	15
Task D	19	26	24	10

| 18 | 26 | 17 | 11 | $P_1 = 11$
| 13 | 28 | 14 | 26 | $P_2 = 13$
| 10 | 40 | 18 | 15 | $P_3 = 10$
| 19 | 26 | 24 | 10 | $P_4 = 10$

7	15	6	0
0	15	1	13
0	30	8	5
9	16	14	0

$q_1 = 0$ $q_2 = 15$ $q_3 = 1$ $q_4 = 0$

7	0	5	0
0	0	0	13
0	15	7	5
9	1	13	0

(blue 1's marking assignments: Person 2→A, Person 3→B, Person 1→C, Person 4→D)

*Optimal!

Solution: $X_{12} = 1$, Person 2 gets Task A
$X_{23} = 1$, Person 3 gets Task B
$X_{31} = 1$, Person 1 gets Task C
$X_{34} = 1$, Person 4 gets Task D

$Z_{opt} = 10 + 26 + 14 + 10 = \boxed{60}$

3. There are four teachers in the John L. Grove College of Business School. Each semester, 200 students [(5)] take each of the following courses: marketing, finance, production, and statistics. The "effectiveness" of each teacher in teaching each class is given in the table below. Each teacher can teach a total of 200 students during the semester. The dean has set a goal of obtaining an average teaching effectiveness of about 6 in each course. Deviations from this goal in any course are considered equally important. Formulate an LP model that can be used to determine the semester's teaching assignments. Solve the problem in Excel. (Hint: Define x_{ij} to be the number of students taking professor i for course j and define a slack and excess variable for each constraint. The goal, then, is to minimize overall slack or overall excess. What should be minimized?)

	$j=1$ Marketing	$j=2$ Finance	$j=3$ Production	$j=4$ Statistics
$i=1$ Teacher 1	7	5	8	2
$i=2$ Teacher 2	7	8	9	4
$i=3$ Teacher 3	3	5	7	9
$i=4$ Teacher 4	5	5	6	7

Decision Variables: x_{ij} = the # of students taking professor i for course j.

Constraints: Note that $\sum_{i=1}^{4} x_{ij} = 200$ for each $j = 1,2,3,4$ and $\sum_{j=1}^{4} x_{ij} = 200$ for all $i = 1,2,3,4$.

(Marketing) $\dfrac{7x_{11} + 7x_{21} + 3x_{31} + 5x_{41}}{x_{11}+x_{21}+x_{31}+x_{41}} \geq 6$ $7x_{11} + 7x_{21} + 3x_{31} + 5x_{41} \geq 1200$

(Finance) $\dfrac{5x_{12} + 8x_{22} + 5x_{32} + 5x_{42}}{x_{12}+x_{22}+x_{32}+x_{42}} \geq 6$ $5x_{12} + 8x_{22} + 5x_{32} + 5x_{42} \geq 1200$

$\Rightarrow$

(Production) $\dfrac{8x_{13} + 9x_{23} + 7x_{33} + 6x_{43}}{x_{13}+x_{23}+x_{33}+x_{43}} \geq 6$ $8x_{13} + 9x_{23} + 7x_{33} + 6x_{43} \geq 1200$

(Statistics) $\dfrac{2x_{14} + 4x_{24} + 9x_{34} + 7x_{44}}{x_{14}+x_{24}+x_{34}+x_{44}} \geq 6$ $2x_{14} + 4x_{24} + 9x_{34} + 7x_{44} \geq 1200$

$x_{ij} \geq 6$ for all $i,j = 1,2,3,4.$

Minimize $z = s_1^- + s_2^- + s_3^- + s_4^-$

Subject to
$$7x_{11} + 7x_{21} + 3x_{31} + 5x_{41} + s_1 - e_1 = 1200$$
$$5x_{12} + 8x_{22} + 5x_{32} + 5x_{42} + s_2 - e_2 = 1200$$
$$8x_{13} + 9x_{23} + 7x_{33} + 6x_{43} + s_3 - e_3 = 1200$$
$$2x_{14} + 4x_{24} + 9x_{34} + 7x_{44} + s_4 - e_4 = 1200$$

Excel Solution: $x_{11} = 200$, $x_{22} = 150$, $x_{33} = 67$, $x_{41} = 153$ $s_1 = 964$ $\min z = 964$
$x_{23} = 50$, $x_{34} = 133$, $x_{43} = 47$