By signing below, you attest that you have neither given nor received help of any kind on this exam.

Signature: Brooks Emerick Printed Name: Brooks Emerick

Instructions: Show work to get full credit (the correct answer may NOT be enough). Do all your work on the paper provided. Write clearly! Double check your answers!

You will **not** receive full credit for using methods other than those discussed in class.

For any LP formulation, you do not have to put it in standard form. You also DO NOT have to organize all of the constraints into a nice square form. I will be grading the formulation on the correctness of the decision variables, objective function, and the constraints, not on the presentation or organization of the constraints.

Exam I

MAT 362 – Operations Research II

Solutions!

Problem Number	Available Points	Your Points
1	9	9
2	6	6
3	10	10
Total	25	25

1. Consider the following LP: [(9)]

$$\text{Minimize:} \quad z = 2x_1 + 3x_2$$

$$\begin{aligned} \text{Subject to:} \quad 2x_1 + 3x_2 &\le 30 \\ x_1 + 2x_2 &\ge 10 \\ x_1 - x_2 &\ge 0 \\ x_1, x_2 &\ge 0 \end{aligned}$$

Determine whether or not the following pairs of primal-dual solutions are optimal:

(a) Find the dual LP. [3]

Maximize $\omega = 30y_1 + 10y_2$

Subject to $2y_1 + y_2 + y_3 \le 2$

$3y_1 + 2y_2 - y_3 \le 3$

$y_1 \le 0,\ y_2 \ge 0,\ y_3 \ge 0$

(b) For each pair of primal and dual solutions below, determine if they are optimal. In either case, optimal or not, use appropriate justification. [6]

i. Primal: $x_1 = 10$, $x_2 = 10/3$. Dual: $y_1 = 0$, $y_2 = 1$, $y_3 = 1$ [2]

- Both solutions are feasible but the objective function values are not equal:

Primal $z = 2(10) + 3(\frac{10}{3}) = 30$

Dual $\omega = 30(0) + 10(1) = 10$

not equal $\Rightarrow$ $\boxed{\text{Not optimal}}$

ii. Primal: $x_1 = 20$, $x_2 = 10$. Dual: $y_1 = 1$, $y_2 = 4$, $y_3 = 0$ [2]

- Neither solution is feasible:

Primal Constraint 1: $2(20) + 3(10) = 70 \not\le 30$

Dual Constraint 1: $2(1) + 4 + 0 = 6 \not\le 2$

Infeasible $\Rightarrow$ $\boxed{\text{Not optimal}}$

iii. Primal: $x_1 = 10/3$, $x_2 = 10/3$. Dual: $y_1 = 0$, $y_2 = 5/3$, $y_3 = 1/3$ [2]

- Both solutions are feasible and the objective function values are equal:

Primal $z = 2(\frac{10}{3}) + 3(\frac{10}{3}) = \frac{50}{3}$

Dual $\omega = 30(0) + 10(\frac{5}{3}) = \frac{50}{3}$

Equal $\Rightarrow$ $\boxed{\text{Optimal}}$

2. Consider the following LP and its optimal tableau below: [(6)]

Maximize: $z = 10x_1 + 4x_2 + 7x_3$

Subject to:
$$3x_1 + x_2 + 2x_3 \le 7$$
$$2x_1 + x_2 + 3x_3 \le 5$$
$$x_1, x_2, x_3 \ge 0$$

Row	Basic	z	x_1	x_2	x_3	s_1	s_2	RHS
0	z	1	0	0	3	2	2	24
1	x_1	0	1	0	−1	1	−1	2
2	x_2	0	0	1	5	−2	3	1

For each sensitivity question below, use information about the dual problem to support your answer.

(a) Assuming the right-hand side of the first constraint represents a resource, what would the optimal objective function value be if only 3 units of this resource were available instead of 7? (Assume the current basis remains optimal.) [2]

Deviation: $7 + \Delta b_1 = 3 \Rightarrow \Delta b_1 = -4$ Dual: $y_1 = 2, y_2 = 2$

Assuming the basis remains optimal, we have

$$\text{New } z = \text{Old } z + y_1 \Delta b_1 = 24 + 2(-4) = \boxed{16}$$

(b) The current value of c_3 is 7. For what values of the coefficient c_3 does the current basis remain optimal? [2]

Let C_3 be the coefficient value, then the third dual constraint becomes $2y_1 + 3y_2 \ge C_3$. The basis remains optimal if the dual solution remains feasible, i.e.

$$2(2) + 3(2) \ge C_3 \Rightarrow \boxed{C_3 \le 10}$$

(c) Suppose we add a new variable, x_4, to the primal problem such that $c_4 = 5$, $a_{14} = 4$ and $a_{24} = -2$. Does the current basis remain optimal? [2]

With this new variable, we have a new dual constraint:

$$4y_1 - 2y_2 \ge 5$$

The basis remains optimal if the dual solution satisfies this constraint.

$$4(2) - 2(2) \overset{?}{\ge} 5 \Rightarrow 4 \not\ge 5$$

The basis is no longer optimal.

3. Consider the unbalanced transportation problem given in the table. If a unit from a source is not shipped out (to any of the destinations), a storage cost is incurred at the rate of \$5, \$4, and \$3 per unit for sources 1, 2, and 3, respectively. Additionally, all the supply at source 2 must be shipped out completely to make room for a new product. [(10)]

Table of Transportation Costs

Sources \ Destinations	1	2	3	Supply
1	\$1	\$2	\$1	20
2	\$3	\$4	\$5	40
3	\$2	\$3	\$3	30
Demand	30	20	20	

(a) Balance the problem by adding a dummy destination and appropriately identify the costs. Then, establish the initial feasible solution using the Northwest-Corner method. [3]

cost 1; 20	cost 2	cost 1	cost 5	20
cost 3; 10	cost 4; 20	cost 5; 10	cost m	40
cost 2	cost 3	cost 3; 10	cost 3; 20	30
30	20	20	20	

(b) Solve the problem with a sequence of iterations using the Transportation Simplex method. Please use the grid tableaus provided. Report your optimal solution and the objective function value below. [7]

$x_{13}=20$, $x_{21}=20$, $x_{22}=20$, $x_{31}=10$, $x_{34}=20$

The rest are zero.

$z_{opt}=240$ (Alternative optima exist.)

	$V_1=1$	$V_2=2$	$V_3=3$	$V_4=3$	
$U_1=0$	$\boxed{1}$ 20	$\boxed{2}$ 0	* $\boxed{1}$ 2	$\boxed{5}$ −2	20
$U_2=2$	$\boxed{3}$ 10	$\boxed{4}$ 20	$\boxed{5}$ 10	$\boxed{M}$ 5−M	40
$U_3=0$	$\boxed{2}$ −1	$\boxed{3}$ −1	$\boxed{3}$ 10	$\boxed{3}$ 20	30
	30	20	20	20	

$\theta=10$

$\boxed{1}$ $20-\theta$	$\boxed{2}$	$\boxed{1}$ θ	$\boxed{5}$	20
$\boxed{3}$ $10+\theta$	$\boxed{4}$ 20	* $\boxed{5}$ $10-\theta$	$\boxed{M}$	40
$\boxed{2}$	$\boxed{3}$	$\boxed{3}$ 10	$\boxed{3}$ 20	30
30	20	20		

	$V_1=1$	$V_2=2$	$V_3=1$	$V_4=1$	
$U_1=0$	$\boxed{1}$ 10	$\boxed{2}$ 0	$\boxed{1}$ 10	$\boxed{5}$ −4	20
$U_2=2$	$\boxed{3}$ 20	$\boxed{4}$ 20	$\boxed{5}$ −2	$\boxed{M}$ 3−M	40
$U_3=2$	* $\boxed{2}$ 1	$\boxed{3}$ 1	$\boxed{3}$ 10	$\boxed{3}$ 20	30
	30	20	20	20	

$\theta = 10$

[1] $10-\theta$	[2]	[1] $10+\theta$	[5]	20
[3] 20	[4] 20	[5]	[m]	40
[2] θ	[3]	[3] x $10-\theta$	[3] 20	30
30	20	20	20	

	$v_1=1$	$v_2=2$	$v_3=1$	$v_4=2$	
$u_1=0$	[1] 0	[2] 0	[1] 20	[5] -3	20
$u_2=2$	[3] 20	[4] 20	[5] -2	[m] $4-m$	40
$u_3=1$	[2] 10	[3] 0	[3] -1	[3] 20	30
	30	20	20	20	

Optimal!

[1]	[2]	[1]	[ ]	20
[3]	[4]	[5]	[ ]	40
[2]	[3]	[3]	[ ]	30
30	20	20		

1	2	1		20
3	4	5		40
2	3	3		30
30	20	20		

1	2	1		20
3	4	5		40
2	3	3		30
30	20	20		

1	2	1		20
3	4	5		40
2	3	3		30
30	20	20		