MAT 106-010 - Trigonometry
Brooks Emerick

CHAPTER 2 - Analytical Trigonometry

Section 2.1: Fundamental Identities

- The most notable identity is Pythagorean's Identity:

(main) $$\cos^2(\theta) + \sin^2(\theta) = 1$$

(version 1) $$1 + \tan^2(\theta) = \sec^2(\theta)$$

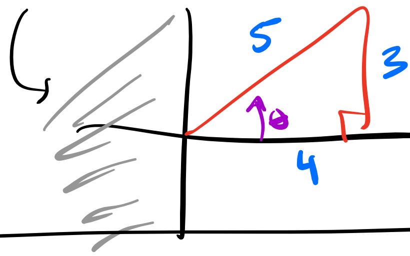
(version 2) $$\cot^2(\theta) + 1 = \csc^2(\theta)$$

- In this section, we use identities to simplify trig expressions or to create new identities.

- WebAssign Due Date now is Wed Nov 16 at 11:59 pm.
  ↳ 1.5, 1.6, P.9, P.10, 1.7, 2.1, 2.2
- Exam III next Friday Nov 18 at 10 AM AF 102

Ex: $$\tan\Big(\underbrace{\arcsin\left(\tfrac{\sqrt{2}}{2}\right)}_{\pi/4}\Big) = \tan(\pi/4) = 1$$

Ex: $\tan(\underbrace{\arcsin(3/5)}_{\sin(\theta)=\frac{3}{5}}) = \tan(\theta) = \boxed{\frac{3}{4}}$



- Wednesday – WebAssign Due 1.5, 1.6, 1.7, P.9, P.10, 2.1, 2.2
- Friday – Test III 1.7, P.9, P.10, 2.1, 2.2 AF 102 10 AM

Section 2.2: Verifying Trig Identities

- For verifying identities:

1.) work with only one side of the equation (the more complicated side) until you stuck.

2.) look for opportunities to factor common elements or create a common denominator.

3.) look for opportunities to use Pythagorean's Identity ($\cos^2(\theta)+\sin^2(\theta)=1$).

4.) If steps 1-3 fail, convert trig expressions to sine and cosine. Use reciprocal identities.

5.) Do something.

Ex: verify the following: $\underbrace{\csc^2(\theta)\tan^2(\theta)}_{LHS} = \underbrace{\sec^2(\theta)}_{RHS}$

LHS: $\csc^2(\theta)\tan^2(\theta) = \dfrac{1}{\sin^2(\theta)} \cdot \tan^2(\theta)$

$$= \frac{1}{\cancel{\sin^2(\theta)}} \cdot \frac{\cancel{\sin^2(\theta)}}{\cos^2(\theta)}$$

$$= \frac{1}{\cos^2(\theta)}$$

$$= \sec^2(\theta) \quad : RHS$$

- Using the graphs of sine, cosine, and tangent, we can come up with the following symmetry identities:

- Even Functions (y-axis symmetric) $f(x) = \cos(x)$ is even

$$\boxed{\cos(\theta) = \cos(-\theta)}$$

- odd Functions! (rotationally symmetric) $f(x) = \sin(x)$ and $f(x) = \tan(x)$ are odd

$$\boxed{-\sin(\theta) = \sin(-\theta)} \quad \boxed{-\tan(\theta) = \tan(-\theta)}$$

Section 2.3: Solving Trigonometric Equations

- Let x be an unknown real number, then solve the trig equation for x:

$$\sin(x) = \frac{1}{2}$$

We're searching for all angles such that sine of that angle results in $\frac{1}{2}$. Answer:

$$x = \frac{\pi}{6},\ \frac{5\pi}{6},\ \underbrace{\frac{\pi}{6}+2\pi}_{\text{coterminal to } \pi/6},\ \underbrace{\frac{5\pi}{6}+2\pi}_{\text{coterminal to } 5\pi/6},\ \underbrace{\frac{\pi}{6}-2\pi}_{\text{coterminal to } \pi/6},\ \frac{5\pi}{6}-2\pi,\ \ldots$$

Obvious answer QI

QII

There's an infinite list of possible answers. We can write our answers in compact form:

$$\boxed{x = \frac{\pi}{6} + n \cdot 2\pi} \qquad \boxed{x = \frac{5\pi}{6} + n \cdot 2\pi}$$

where n is any integer. $\{\ldots -3, -2, -1, 0, 1, 2, \ldots\}$

Ex: Suppose $x \in [0, 2\pi)$ (within one positive rotation). Now solve

$$\sin(x) = \frac{1}{2}$$

Solution: $\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$ Only answers within one positive revolution

Ex: Suppose $x \in [0, 2\pi)$. Solve

$$2\sin(x) - 1 = 0$$

$$2\sin(x) = 1$$

$$\sin(x) = \frac{1}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$$

$2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

Section 2.4: Sum & Difference Identities

- Sum & Difference Identities: let u and v be any two angles.

  - $\sin(u+v) = \sin(u)\cos(v) + \cos(u)\sin(v)$
  - $\sin(u-v) = \sin(u)\cos(v) - \cos(u)\sin(v)$

  - $\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$
  - $\cos(u-v) = \cos(u)\cos(v) + \sin(u)\sin(v)$

Ex: Calculate $\sin(75^\circ)$ exactly.

$$\sin(75^\circ) = \sin(\overset{u}{45^\circ} + \overset{v}{30^\circ})$$

$$= \sin(45^\circ)\cos(30^\circ) + \cos(45^\circ)\sin(30^\circ)$$

$$= \sin(\pi/4)\cos(\pi/6) + \cos(\pi/4)\sin(\pi/6)$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$\sin(75^\circ) = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

$$\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12}$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

Ex: Calculate $\cos(\pi/12)$ exactly.

$$\cos(\pi/12) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \cos(\pi/3)\cos(\pi/4) + \sin(\pi/3)\sin(\pi/4)$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

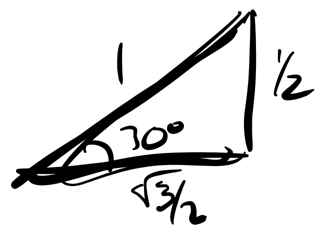
$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

• Sum / Difference Identities for tangent:

$$\bullet \ \tan(u+v) = \frac{\tan(u)+\tan(v)}{1-\tan(u)\tan(v)}$$

$$\bullet \ \tan(u-v) = \frac{\tan(u)-\tan(v)}{1+\tan(u)\tan(v)}$$



Ex: $\tan(75^\circ) = \tan(30^\circ + 45^\circ)$

$$= \frac{\tan(30^\circ) + \tan(45^\circ)}{1-\tan(30^\circ)\tan(45^\circ)}$$

$$= \frac{1/\sqrt{3} + 1}{1 - \left(\frac{1}{\sqrt{3}}\right)(1)}$$

$$= \boxed{\frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}}} = \frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \boxed{\frac{1+\sqrt{3}}{\sqrt{3}-1}}$$

Quiz 10: Wednesday, Dec 7, In class

WebAssign: Friday, Dec 9, 11:59 PM.

Final Exam: Monday, Dec 12, 8:00 AM, AF 102

→ ~30% is on 2.2 – 2.5

~70% is from previous stuff.

→ Study Past Exams.

Section 2.5: Double-Angle Identities, Power-Reducing Identities, and Product-to-Sum Identities

• Double-Angle Formulas:

- $\sin(2x) = 2\sin(x)\cos(x)$

- $\cos(2x) = \cos^2(x) - \sin^2(x)$

$= 2\cos^2(x) - 1$

$= 1 - 2\sin^2(x)$

- $\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$

Final Exam: 12 questions

AF 102 • 5 are from 2.2 - 2.5

8AM • 7 are from stuff from P, 1, 2.1

(Donut Holes!)

Section 2.5: Continued

- Power-Reducing Identities:

  - $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
  - $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
  - $\tan^2(x) = \dfrac{1-\cos(2x)}{1+\cos(2x)}$

x is any angle

Product-to-Sum Identities:

- $\sin(u)\sin(v) = \frac{1}{2}\cos(u-v) - \frac{1}{2}\cos(u+v)$
- $\cos(u)\cos(v) = \frac{1}{2}\cos(u-v) + \frac{1}{2}\cos(u+v)$

- $\sin(u)\cos(v) = \frac{1}{2}\sin(u+v) + \frac{1}{2}\sin(u-v)$
- $\cos(u)\sin(v) = \frac{1}{2}\sin(u+v) - \frac{1}{2}\sin(u-v)$