§1.1-1.3: Functions and Stuff

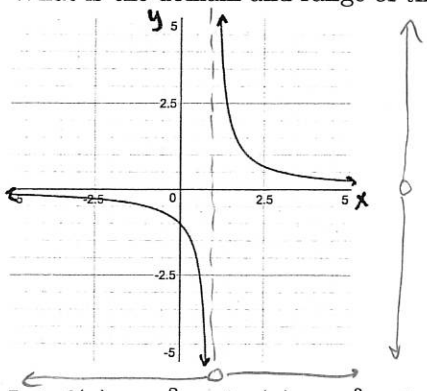
1.] Find the domain of the function $f(x) = \dfrac{\sqrt{24 - 6x^2}}{2x - 1}$.

$24 - 6x^2 \geq 0 \Rightarrow 24 \geq 6x^2 \Rightarrow x^2 \leq 4 \Rightarrow \sqrt{x^2} \leq 2 \Rightarrow |x| \leq 2 \Rightarrow \boxed{-2 \leq x \leq 2}$

$2x - 1 \neq 0 \Rightarrow 2x \neq 1 \Rightarrow \boxed{x \neq \frac{1}{2}}$

Domain: $\boxed{[-2, \tfrac{1}{2}) \cup (\tfrac{1}{2}, 2]}$

2.] What is the domain and range of the function depicted in the graph below:



Domain: $x \neq 1$ or $(-\infty, 1) \cup (1, \infty)$

Range: $y \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

3.] Let $f(x) = x^2$ and $g(x) = x^3 + 1$. Find an expression for $f(g(x))$ and $g(f(x))$. Are they the same function?

$(f \circ g)(x) = f(g(x)) = f(x^3+1) = (x^3+1)^2 = (x^3)^2 + 2(x^3)(1) + (1)^2 = \boxed{x^6 + 2x^3 + 1}$

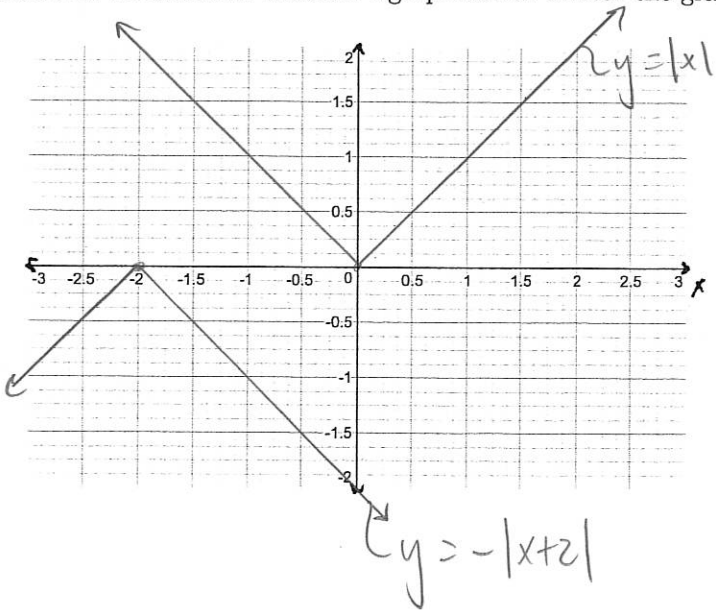
$(g \circ f)(x) = g(f(x)) = g(x^2) = (x^2)^3 + 1 = \boxed{x^6 + 1}$ ——— not the same.

4.] Represent the function $f(x) = |x|$ as a piecewise function and sketch it's graph below. Sketch the graph of $g(x) = -|x + 2|$ as well.

$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$g(x) = -|x+2|$

- the "+2" shifts the graph left two units
- the "−" flips the graph about the x-axis.



5.] Find the linear function f that goes through the points $(0, 6)$ and $(2, 10)$. Is f one-to-one? If so, find the inverse function f^{-1}. Show that $f(f^{-1}(x)) = x$.

Linear Function

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-6}{2-0} = 2$$

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 10 = 2(x-2)$$

$$\Rightarrow y = 2x + 6$$

$$\Rightarrow \boxed{f(x) = 2x+6}$$

Inverse Function

$$y = 2x + 6$$

$$\Rightarrow x = 2y + 6$$

$$\Rightarrow 2y = x - 6$$

$$\Rightarrow y = \frac{1}{2}x - 6$$

$$\Rightarrow \boxed{f^{-1}(x) = \frac{1}{2}x - 3}$$

Sanity Check

$$f(f^{-1}(x)) = f\left(\frac{1}{2}x - 6\right)$$

$$= 2\left(\frac{1}{2}x - 3\right) + 6$$

$$= x - 6 + 6$$

$$= x \quad \checkmark$$

6.] Solve the following equation for x: $\log_8(x) = \frac{1}{3}$

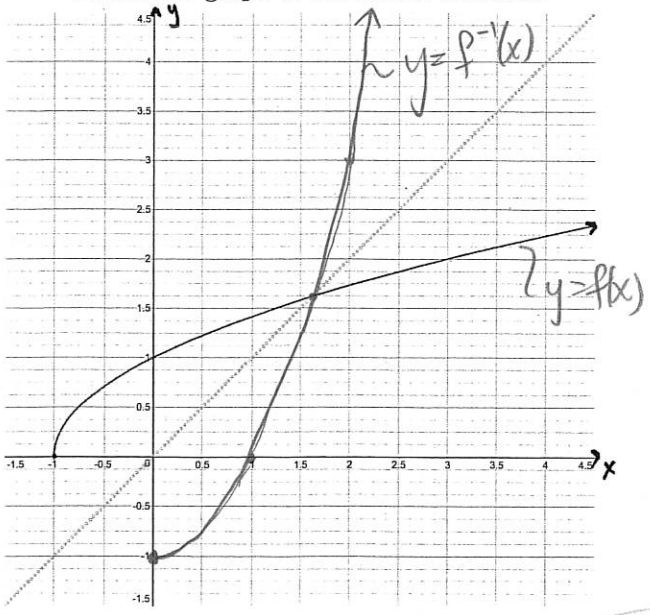
$$\log_8(x) = \frac{1}{3} \Rightarrow 8^{\log_8(x)} = 8^{1/3} \Rightarrow x = 8^{1/3} \Rightarrow \boxed{x = 2}$$

7.] Solve the following equation for x: $3^{3x-4} = 5$

$$3^{3x-4} = 5 \Rightarrow \ln(3^{3x-4}) = \ln(5) \Rightarrow (3x-4)\ln(3) = \ln(5) \Rightarrow 3x - 4 = \frac{\ln(5)}{\ln(3)}$$

$$\Rightarrow \boxed{x = \frac{1}{3}\left(\frac{\ln(5)}{\ln(3)} - 4\right)}$$

8.] The function sketched below is $f(x) = \sqrt{x+1}$. Find the domain of $f(x)$ and the domain of $f^{-1}(x)$. Sketch the graph of the inverse function.



Domain of f: $[-1, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[-1, \infty)$

$$y = \sqrt{x+1} \Rightarrow x = \sqrt{y+1} \Rightarrow x^2 = y + 1$$

$$\hookrightarrow \Rightarrow y = x^2 - 1 \Rightarrow \boxed{f^{-1}(x) = x^2 - 1}$$