§P.9 & P.10: Function Composition and Inverse Functions

1.] Let $f(x) = \sqrt{x+4}$ and $g(x) = x^2$. Find expressions for the new functions below:

a.) $(f \circ g)(x) = f(g(x)) = f(x^2) = \boxed{\sqrt{x^2+4}}$

b.) $(g \circ f)(x) = g(f(x)) = g(\sqrt{x+4}) = (\sqrt{x+4})^2 = \boxed{x+4}$

2.] Let $f(x) = \dfrac{1}{x}$ and $g(x) = x^2 + 3$. Find expressions for the new functions below:

a.) $(f \circ g)(x) = f(g(x)) = f(x^2+3) = \boxed{\dfrac{1}{x^2+3}}$

b.) $(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 + 3 = \boxed{\frac{1}{x^2} + 3}$

c.) $(g \circ g)(x) = g(g(x)) = g(x^2+3) = (x^2+3)^2 + 3 = \boxed{x^4 + 6x^2 + 12}$

d.) $(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \dfrac{1}{\frac{1}{x}} = \boxed{x}$

3.] For each function $h(x)$ below, find two functions $f(x)$ and $g(x)$ such that $h(x) = (f \circ g)(x)$.

a.) $h(x) = (2x+1)^2$

$f(x) = x^2$, $g(x) = 2x+1$ then $f(g(x)) = f(2x+1) = (2x+1)^2 = h(x)$!

b.) $h(x) = \sqrt[3]{x^2 - 4}$

$f(x) = \sqrt[3]{x-4}$
$g(x) = x^2$ then $f(g(x)) = f(x^2) = \sqrt[3]{x^2-4} = h(x)$!

4.] Find the inverse function of $f(x) = 3x + 1$ and verify that $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$.

$y = 3x + 1$

$\Rightarrow x = 3y + 1$

$\Rightarrow 3y = x - 1$

$\Rightarrow y = \frac{1}{3}x - \frac{1}{3}$

$\Rightarrow \boxed{f^{-1}(x) = \frac{1}{3}x - \frac{1}{3}}$

$f(f^{-1}(x)) = f\left(\frac{1}{3}x - \frac{1}{3}\right) = 3\left(\frac{1}{3}x - \frac{1}{3}\right) + 1 = x - 1 + 1 = x$ ✓

$f^{-1}(f(x)) = f^{-1}(3x+1) = \frac{1}{3}(3x+1) - \frac{1}{3} = x + \frac{1}{3} - \frac{1}{3} = x$

5.] Find the inverse function of $f(x) = 1 - x^3$.

$y = 1 - x^3$

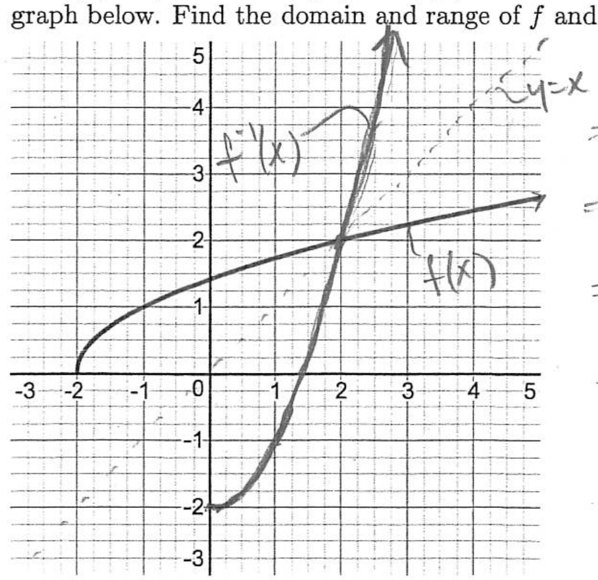
$\Rightarrow x = 1 - y^3$

$\Rightarrow y^3 = 1 - x$

$\Rightarrow y = \sqrt[3]{1-x}$

$\boxed{f^{-1}(x) = \sqrt[3]{1-x}}$

6.] Let $f(x) = \sqrt{2+x}$. The graph of $f(x)$ is below. Find the inverse function $f^{-1}(x)$ and sketch it on the graph below. Find the domain and range of f and f^{-1}.



$y = \sqrt{2+x}$

$\Rightarrow x = \sqrt{2+y}$

$\Rightarrow x^2 = 2 + y$

$\Rightarrow y = x^2 - 2$

$\Rightarrow f^{-1}(x) = x^2 - 2$

Domain of f: $[-2, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[-2, \infty)$