§P.5: Functions

1.] Determine whether the relations (presented with a table of values) represent y as a function of x.

Input, x	10	7	4	7	10
Output, y	3	6	9	12	15

Not a function because the input value $x=10$ has two different outputs: $y=3$ and $y=15$. Same with $x=7$.

Input, x	−2	0	2	4	6
Output, y	1	1	1	1	1

This is a function because all input values of x correspond to a single y-value, $y=1$, even though all outputs are the same.

2.] Let $f(x) = 4x^2 - 3x + 5$. Find each function value below:

a.) $f(2)$

$f(2) = 4(2)^2 - 3(2) + 5$

$\Rightarrow f(2) = 4(4) - 6 + 5$

$\Rightarrow f(2) = 16 - 6 + 5$

$\boxed{f(2) = 15}$

b.) $f(x-2)$

$f(x-2) = 4(x-2)^2 - 3(x-2) + 5$

$\Rightarrow f(x-2) = 4(x-2)(x-2) - 3x + 6 + 5$

$\Rightarrow f(x-2) = 4(x^2 - 2x - 2x + 4) - 3x + 11$

$\Rightarrow f(x-2) = 4(x^2 - 4x + 4) - 3x + 11$

$f(x-2) = 4x^2 - 16x + 16 - 3x + 11$

$\boxed{f(x-2) = 4x^2 - 19x + 27}$

c.) $f(x) - f(2)$

$f(x) - f(2) = (4x^2 - 3x + 5) - (15)$

$\boxed{f(x) - f(2) = 4x^2 - 3x - 10}$

3.] Let $f(x) = \sqrt{x} - 4$ and $g(x) = 2 - x$. Determine all x values where $f(x) = g(x)$.

$f(x) = g(x)$

$\Rightarrow \sqrt{x} - 4 = 2 - x$

$\Rightarrow \sqrt{x} = 6 - x$

$\Rightarrow (\sqrt{x})^2 = (6-x)^2$

$\Rightarrow x = (6-x)(6-x)$

$\Rightarrow x = 36 - 12x + x^2$

$\Rightarrow x^2 - 13x + 36 = 0$

$\Rightarrow (x-9)(x-4) = 0$

$\Rightarrow x - 9 = 0 \qquad x - 4 = 0$

$\cancel{x=9} \qquad \boxed{x=4}$

Extraneous!!

4.] What is the domain of the function $f(x) = 1 - 2x^2$

We can plug any real number into f and get a real number back i.e. $f(x)=1-2x^2$ is defined everywhere. $\boxed{\text{Domain: } \mathbb{R} \text{ or } (-\infty,\infty)}$

5.] What is the domain of the function $f(x) = \sqrt{3x-5}$

We cannot take the square root of a negative number so we must have

$3x-5 \geq 0 \Rightarrow 3x \geq 5 \Rightarrow x \geq 5/3$ $\boxed{\text{Domain: } [5/3, \infty)}$

6.] What is the domain of the function $f(x) = \sqrt[3]{x+4}$

It is legal to take the cubed root of negative and positive numbers, so all real numbers can be inputted into f.

$\boxed{\text{Domain: } \mathbb{R} \text{ or } (-\infty,\infty)}$

7.] What is the domain of the function $f(x) = \dfrac{1}{x^2-4x}$

We cannot divide by zero, so we must make sure the denominator is not zero.

$x^2-4x \neq 0 \Rightarrow x(x-4) \neq 0 \Rightarrow x \neq 0, x \neq 4$ $\boxed{\text{Domain: } (-\infty,0)\cup(0,4)\cup(4,\infty)}$

8.] What is the domain of the function $f(x) = \dfrac{\sqrt{x+2}}{x-10}$

Numerator: $x+2 \geq 0 \Rightarrow x \geq -2$

Denominator: $x-10 \neq 0 \Rightarrow x \neq 10$

$\boxed{\text{Domain: } [-2,10)\cup(10,\infty)}$

9.] What is the domain of the function $f(x) = \dfrac{x+2}{\sqrt{x-10}}$

Numerator: it's fine, no issue

Denominator: $x-10 > 0 \Rightarrow x > 10$

$\boxed{\text{Domain: } (10,\infty)}$

10.] What is the domain of the function $f(x) = \sqrt{4-x^2}$

$4-x^2 \geq 0 \Rightarrow x^2 \leq 4 \Rightarrow -2 \leq x \leq 2$

$\boxed{\text{Domain: } [-2,2]}$