§5.5: u-SUBSTITUTION

1.] Compute the following anti-derivative: $\int \underbrace{3x^2 (x^3+1)^{90}}_{u^{90}} \overbrace{dx}^{du}$

$u = x^3+1$
$du = 3x^2\,dx$

$$\int 3x^2(x^3+1)^{90}\,dx = \int u^{90}\,du = \frac{1}{91}u^{91} + C$$
$$= \boxed{\frac{1}{91}(x^3+1)^{91} + C}$$

2.] Compute the following indefinite integral: $\int \underbrace{\sin^3(x)}_{u^3} \underbrace{\cos(x)\,dx}_{du}$

$u = \sin(x)$
$du = \cos(x)dx$

$$\int \sin^3(x)\cos(x)\,dx = \int u^3\,du = \frac{1}{4}u^4 + C$$
$$= \boxed{\frac{1}{4}\sin^4(x) + C}$$

3.] Compute the following anti-derivative: $\int \frac{x^2}{\sqrt{9+4x^3}}\,dx$

$u = 9+4x^3$
$du = 12x^2dx \rightarrow x^2dx = \frac{1}{12}du$

$$\int \frac{x^2}{\sqrt{9+4x^3}}\,dx = \int \frac{1}{\sqrt{u}}\frac{1}{12}\,du$$
$$= \frac{1}{12}\cdot\frac{1}{1/2}u^{1/2} + C$$
$$= \boxed{\frac{1}{6}(9+4x^3)^{1/2} + C}$$

4.] Compute the following indefinite integral: $\int \frac{\csc^2(x)}{\cot^3(x)}$ dx

$u = \cot(x)$
$du = -\csc^2(x)dx$
$\hookrightarrow \csc^2(x)dx = -du$

$$\int \frac{\csc^2(x)}{\cot^3(x)}\,dx = \int \frac{-1}{u^3}\,du$$
$$= -\int u^{-3}\,du$$
$$= -\frac{1}{-2}u^{-2} + C$$
$$= \boxed{\frac{1}{2\cot^2(x)} + C}$$

5.] Compute the following indefinite integral: $\int x\sqrt[3]{x+1}\,dx$

$u = x+1 \rightarrow x = u-1$

$du = dx$

$$\int x\sqrt[3]{x+1}\,dx = \int (u-1)\sqrt[3]{u}\,du$$
$$= \int u^{4/3} - u^{1/3}\,du$$
$$= \frac{3}{7}u^{7/3} - \frac{3}{4}u^{4/3} + C = \boxed{\frac{3}{7}(x+1)^{7/3} - \frac{3}{4}(x+1)^{4/3} + C}$$

6.] Compute the following anti-derivative: $\int \frac{x}{\sqrt{4-x}}\,dx$

$u = 4-x \rightarrow x = 4-u$

$du = -dx \rightarrow dx = -du$

$$\int \frac{x}{\sqrt{4-x}}\,dx = -\int \frac{4-u}{\sqrt{u}}\,du$$
$$= -\int 4u^{-1/2} - u^{1/2}\,du$$
$$= -4(2u^{1/2}) + \frac{2}{3}u^{3/2} + C$$
$$= \boxed{-8\sqrt{4-x} + \frac{2}{3}(4-x)^{3/2} + C}$$

7.] Compute the following area: $\int_{-1}^{2} x^2 e^{x^3+1}\,dx$

$u = x^3+1$

$du = 3x^2\,dx \rightarrow x^2 dx = \frac{1}{3}du$

$x = -1 \rightarrow u = (-1)^3+1 = 0$

$x = 2 \rightarrow u = 2^3+1 = 9$

$$\int_{-1}^{2} x^2 e^{x^3+1}\,dx = \int_0^9 \frac{1}{3}e^u\,du$$
$$= \frac{1}{3}e^u\Big|_0^9 = \frac{1}{3}e^9 - \frac{1}{3}e^0 = \boxed{\frac{1}{3}(e^9-1)}$$

8.] Compute the following definte integral: $\int_1^{e^2} \frac{\ln(x)}{x}\,dx$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$x = 1 \rightarrow u = \ln(1) = 0$

$x = e^2 \rightarrow u = \ln(e^2) = 2$

$$\int_1^{e^2} \frac{\ln(x)}{x}\,dx = \int_0^2 u\,du = \frac{1}{2}u^2\Big|_0^2$$
$$= \frac{1}{2}(2)^2 - \frac{1}{2}(0)^2$$
$$= \boxed{2}$$