§4.3: Graphing functions

1.] Make a complete graph of the function $f(x) = x^4 - 6x^2$. Identify local extrema, inflection points, and $x-$ and $y-$ intercepts.

- Domain: $\mathbb{R}$
- $f'(x) = 4x^3 - 12x = 4x(x^2-3) = 4x(x+\sqrt{3})(x-\sqrt{3})$

  Critical points: $c=-\sqrt{3}$, $c=0$, $c=\sqrt{3}$
- $f''(x) = 12x^2 - 12 = 12(x^2-1) = 12(x+1)(x-1)$

  Poss. Inflection Pts: $x=-1$, $x=1$

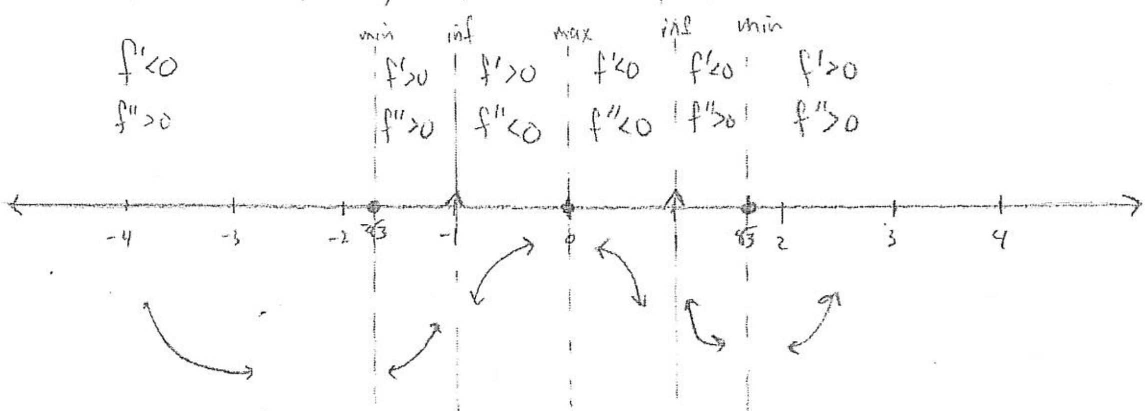
Local Mins: $(-\sqrt{3},-9)$ and $(\sqrt{3},-9)$

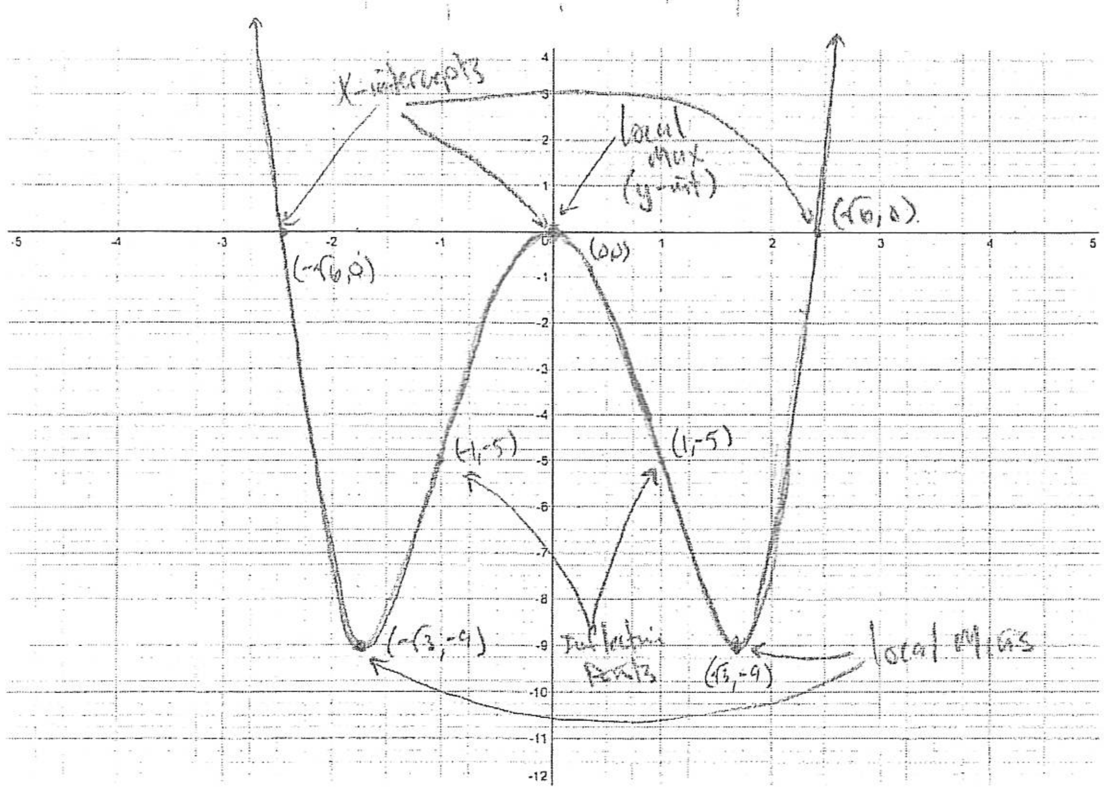
Local Max: $(0,0)$

Inflection Pts: $(-1,-5)$ and $(1,-5)$

x-int: $x^4 - 6x^2 = 0 \Rightarrow x^2(x^2-6)=0$, $x=0, \sqrt{6}, -\sqrt{6}$

y-int: $f(0)=0 \Rightarrow (0,0)$





2.] Make a complete graph of the function $f(x) = x - 3x^{1/3}$. Identify local extrema, inflection points, and $x-$ and $y-$intercepts.

$f(x) = x - 3x^{1/3} \Rightarrow x - 3x^{1/3} = 0 \Rightarrow x^{1/3}(x^{2/3} - 3) = 0 \Rightarrow \boxed{x = 0,\ x = -3\sqrt{3},\ 3\sqrt{3}}$

$f'(x) = 1 - \frac{1}{x^{2/3}} = \frac{x^{2/3} - 1}{x^{2/3}} = 0 \Rightarrow \boxed{C = -1,\ C = 0,\ C = 1}$ — Critical points * $f' = DNE$ at $c = 0$

$f''(x) = \frac{2}{3}\frac{1}{x^{5/3}} \Rightarrow f'' = DNE$ at $\boxed{x = 0}$ — Possible Inflection Point.

- x-int: $(0,0)$, $(-3\sqrt{3},0)$, $(3\sqrt{3},0)$
- y-int: $(0,0)$
- Min: $(1,-2)$
- Max: $(-1,2)$
- Inflection: $(0,0)$