§4.9: ANTIDERIVATIVES

1.] Find the antiderivative of the following functions:

$a.)\ f(x) = 100x^{99}$

$\boxed{F(x) = x^{100} + C}$

$b.)\ g(x) = -\dfrac{1}{\sqrt{1-x^2}}$

$\boxed{G(x) = \arccos(x) + C}$

or $G(x) = -\arcsin(x) + C$

$c.)\ h(x) = \sec(x)\tan(x)$

$\boxed{H(x) = \sec(x) + C}$

2.] Compute the following indefinite integrals:

$a.)\ \int 20x^{19}\,dx$

$= \boxed{x^{20} + C}$

$b.)\ \int -\csc^2(x)\,dx$

$= \boxed{\cot(x) + C}$

$c.)\ \int \frac{1}{x}\,dx$

$= \boxed{\ln(x) + C}$

3.] Use the sum/difference and constant multiple rule for integration to compute the following indefinite integrals:

$a.)\ \int (4x^3 + 1 + \cos(x))\,dx$

$= \int 4x^3 dx + \int 1 dx + \int \cos(x) dx$

$= (x^4 + C) + (x + C) + (\sin(x) + C)$

$= \boxed{x^4 + x + \sin(x) + C}$

$b.)\ \int (x^2 + 3x + \sin(x))\,dx$

$= \int x^2 dx + \int 3x\, dx + \int \sin(x) dx$

$= \frac{3}{3}\int x^2 dx + \frac{2}{2}\int 3x\, dx + -\int(-\sin(x))dx$

$= \frac{1}{3}\int 3x^2 dx + \frac{3}{2}\int 2x\,dx - \int(-\sin(x))dx$

$= \boxed{\frac{1}{3}x^3 + \frac{3}{2}x^2 - \cos(x) + C}$

$c.)\ \int 2^x\,dx$

$= \frac{\ln(2)}{\ln(2)}\int 2^x dx$

$= \frac{1}{\ln(2)}\int \ln(2)\, 2^x dx$

$= \boxed{\frac{1}{\ln(2)} 2^x + C}$

4.] Use the power rule for integration to compute the following indefinite integrals:

a.) $\int \left(3x^5 + 2 - 5x^{-2/3}\right) dx$

$= 3\int x^5\,dx + 2\int 1\,dx - 5\int x^{-2/3}\,dx$

$= 3\left(\frac{1}{6}x^6\right) + 2(x) - 5\left(\frac{1}{1/3}x^{1/3}\right)$

$= \boxed{\frac{1}{2}x^6 + 2x - 15x^{1/3} + C}$

b.) $\int (x^2+1)(2x-5)\,dx$

$= \int (2x^3 - 5x^2 + 2x - 5)\,dx$

$= \frac{2}{4}x^4 - \frac{5}{3}x^3 + x^2 - 5x + C$

$= \boxed{\frac{1}{2}x^4 - \frac{5}{3}x^3 + x^2 - 5x + C}$

c.) $\int \left(\frac{4x^{19} - 5x^{-8}}{x^2}\right) dx$

$= \int (4x^{17} - 5x^{-10})\,dx$

$= \frac{4}{18}x^{18} - \left(\frac{5}{-9}\right)x^{-9} + C$

$= \boxed{\frac{2}{9}x^{18} + \frac{5}{9}x^{-9} + C}$

5.] Solve the initial value problem given by

$$f'(x) = 7x\left(x^6 - \frac{1}{7}\right) \qquad f(1) = 2.$$

$\Rightarrow \int f'(x)\,dx = \int 7x\left(x^6 - \frac{1}{7}\right) dx$

$\Rightarrow f(x) = \int (7x^7 - x)\,dx$

$\Rightarrow f(x) = \frac{7}{8}x^8 - \frac{1}{2}x^2 + C$

$f(1) = \frac{7}{8}(1)^8 - \frac{1}{2}(1)^2 + C$

$\Rightarrow 2 = \frac{7}{8} - \frac{1}{2} + C$

$\Rightarrow 2 = \frac{3}{8} + C$

$\Rightarrow \frac{13}{8} = C$

$\Rightarrow \boxed{f(x) = \frac{7}{8}x^8 - \frac{1}{2}x^2 + \frac{13}{8}}$

6.] Suppose the acceleration function of an object moving along a line is given by $a(t) = 0.2t$. Find the position function of the object if you know the initial velocity was $v(0) = -3$ and initial position was $s(0) = 1$.

$a(t) = 0.2t$

$\Rightarrow v'(t) = 0.2t$

$\Rightarrow \int v'(t)\,dt = \int 0.2t\,dt$

$\Rightarrow v(t) = \frac{0.2}{2}t^2 + C$

$\Rightarrow v(t) = (0.1)t^2 + C$

$\Rightarrow v(0) = (0.1)(0)^2 + C$

$\Rightarrow -3 = C$

$\Rightarrow \boxed{v(t) = (0.1)t^2 - 3}$

$v(t) = (0.1)t^2 - 3$

$\Rightarrow s'(t) = 0.1t^2 - 3$

$\Rightarrow \int s'(t)\,dt = \int (0.1t^2 - 3)\,dt$

$\Rightarrow s(t) = \frac{0.1}{3}t^3 - 3t + C$

$\Rightarrow s(0) = \frac{0.1}{3}(0)^3 - 3(0) + C$

$\Rightarrow 1 = C$

$\Rightarrow \boxed{s(t) = \frac{1}{30}t^3 - 3t + 1}$