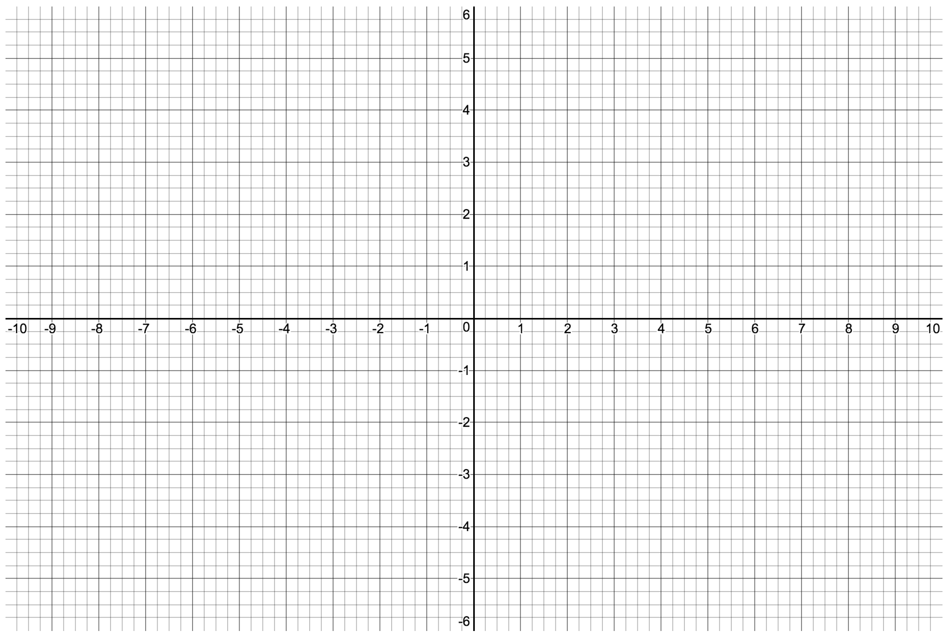
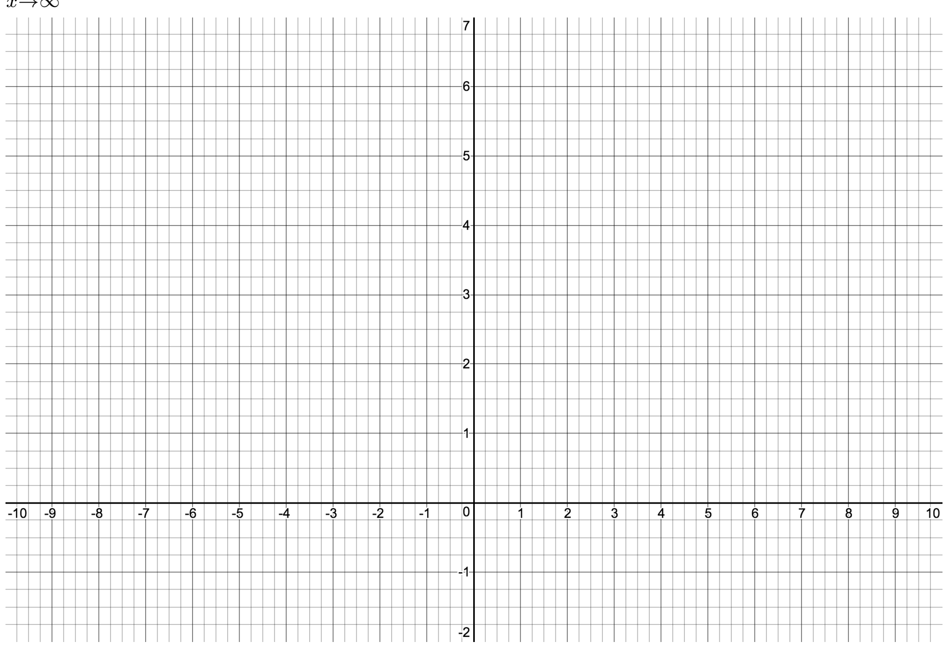
§4.2 (PART 2): CONCAVITY AND SECOND DERIVATIVE TEST

1.] Let $f(x)$ be a continuous function on $(-\infty, \infty)$. Sketch a graph of f that satisfies the following conditions:

a.) $f'(x) < 0$ and $f''(x) < 0$ on $(-\infty, 0)$; $f(0) = 1$; $f'(x) > 0$ and $f''(x) > 0$ on $(0, \infty)$.



b.) $f(x) > 0$ for all x; $f'(x) > 0$ for all x; $f(0) = 2$; $f''(x) > 0$ on $(-\infty, 2)$; $f''(x) < 0$ on $(2, \infty)$; $\lim_{x \to \infty} f(x) = 5$.



2.] Determine the intervals on which the function $f(x) = 3x^5 - 30x^4 + 80x^3 + 100$ are concave up or concave down. Identify all inflection points.

3.] Locate the critical points of the function $f(x) = x^2 e^{-x}$. Then use the Second Derivative Test to determine whether the critical points correspond to local maxima, local minima, or neither.