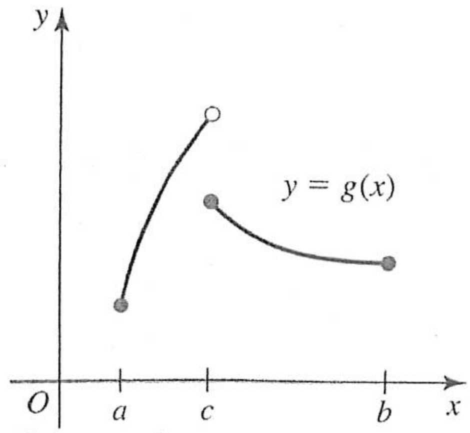
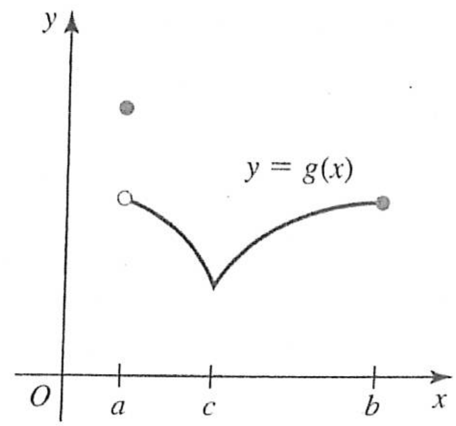
# §4.1: Maxima, Minima, and Critical Points

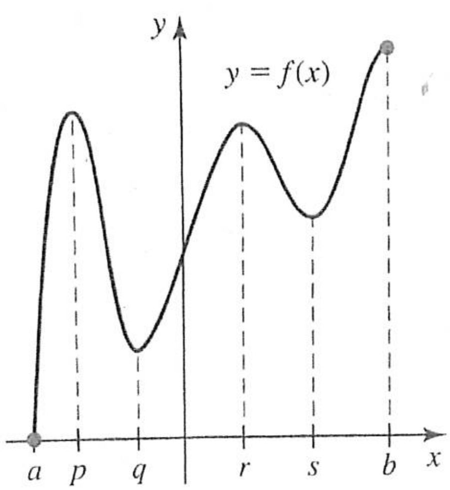
1.] In the graphs below, identify the points (if any) on the interval $[a, b]$ at which the function has an absolute max, absolute min, local min, or local max.



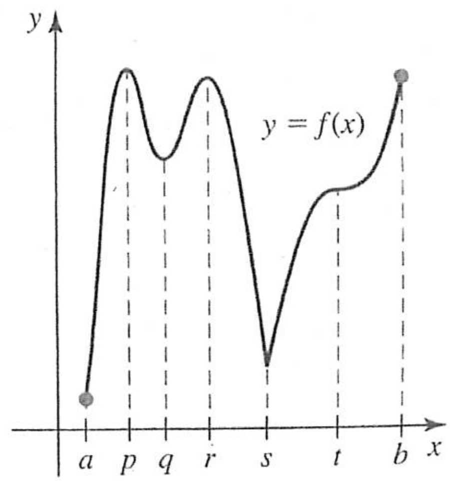

Abs min at $x=a$.
No abs min
No local extrema




Abs max at $x=a$
Abs min at $x=c$
local min at $x=c$
No local max




Abs min at $x=a$
Abs max at $x=b$
local max at $x=p, x=r$
local min at $x=q, x=s$




Abs min at $x=a$
Abs max at $x=p$
local max at $x=p, x=r$
local min at $x=q, x=s$

2.] Find the critical points of the following functions on the domain or on the given interval.

a.) $f(x) = \frac{x^3}{3} - 9x$ on $[-7, 7]$

$f'(x) = x^2 - 9$

$f'(x) = \text{DNE}?$

N/A

$f'(x) = 0?$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

Since $-3$ and $3$ are both inside $[-7,7]$, we have two critical points:

$\boxed{c_1 = -3,\ c_2 = 3}$

b.) $f(x) = x^2\sqrt{x+5}$ Domain: $[0, \infty)$

$f'(x) = 2x \cdot \sqrt{x+5} + x^2\left(\frac{1}{2\sqrt{x+5}} \cdot 1\right)$

$f'(x) = 2x \cdot \sqrt{x+5} + \frac{x^2}{2\sqrt{x+5}}$

$f'(x) = \frac{4x(x+5) + x^2}{2\sqrt{x+5}}$

$f'(x) = \frac{5x^2 + 20x}{2\sqrt{x+5}}$

$f'(x) = \text{DNE}?$

$\sqrt{x+5} = 0$

$x = -5$

$f'(x) = 0?$

$5x^2 + 20x = 0$

$5x(x+4) = 0$

$x = 0, -4$

Critical pts:

$\boxed{\begin{matrix} c_1 = -4 \\ c_2 = 0 \end{matrix}}$

* $-5$ is not critical, not inside $[0, \infty)$

3.] Find the critical points of the function $f(x) = 3x^5 - 25x^3 + 60x$ on the interval $[-2, 3]$. Determine the absolute extreme values of $f$ on the given interval.

$f'(x) = 15x^4 - 75x^2 + 60$

$f'(x) = 15(x^4 - 5x^2 + 4)$

$f'(x) = 15(x^2 - 4)(x^2 - 1)$

$f'(x) = 15(x-2)(x+2)(x-1)(x+1)$

$f'(x) = \text{DNE}?$

N/A

$f'(x) = 0?$

$x = -2, -1, 1, 2$

Critical pts:

$\boxed{\begin{matrix} c_1 = -1 \\ c_2 = 1 \\ c_3 = 2 \end{matrix}}$

Closed Interval Method

$f(-2) = 3(-2)^5 - 25(-2)^3 + 60(-2) = -16$

$\boxed{f(-1)} = 3(-1)^5 - 25(-1)^3 + 60(-1) = -38$

$f(1) = 3(1)^5 - 25(1)^3 + 60(1) = 38$

$f(2) = 3(2)^5 - 25(2)^3 + 60(2) = 16$

$\boxed{f(3)} = 3(3)^5 - 25(3)^3 + 60(3) = 234$

Abs min at $x = -1$

Abs max at $x = 3$