§4.2 (PART 1): INCREASING/DECREASING AND FIRST DERIVATIVE TEST

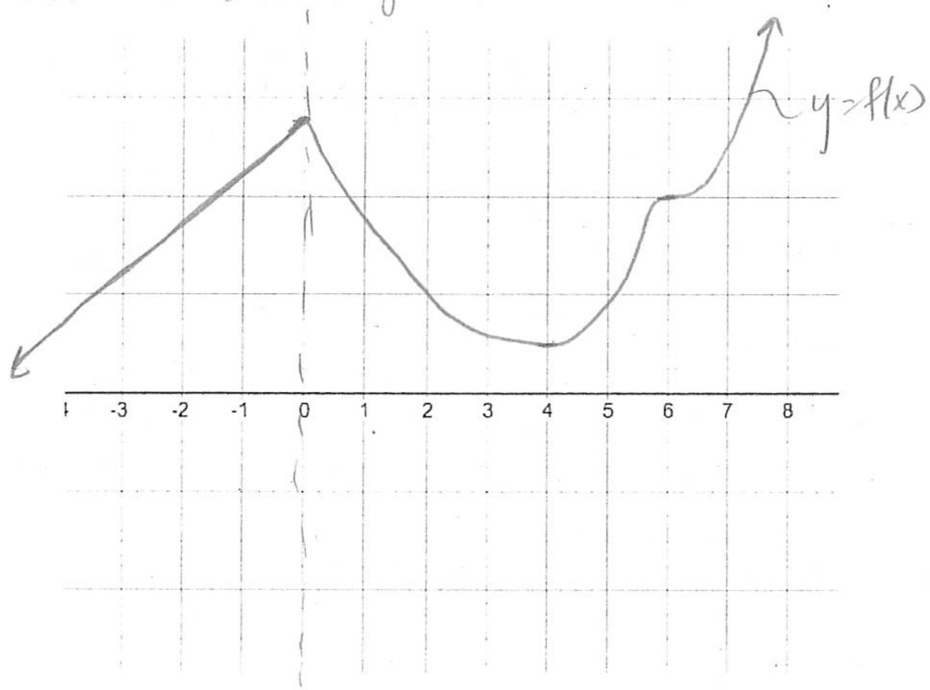
1.] Sketch a function f that is continuous on $(-\infty, \infty)$ and satisfies the following conditions:

a.) $f'(x) > 0$ for x in the intervals $(-\infty, 0)$, $(4, 6)$, and $(6, \infty)$. increasing

b.) $f'(x) < 0$ for x in the interval $(0, 4)$. decreasing

c.) $f'(0)$ is not defined. cusp since f is cont. everywhere.

d.) $f'(4) = f'(6) = 0$. horizontal tangent line



2.] Determine the intervals on which the function $f(x) = x^2 e^{-x/2}$ is increasing and decreasing.

$$f'(x) = x^2\left(-\frac{1}{2}e^{-x/2}\right) + (2x)e^{-x/2}$$

$$f'(x) = xe^{-x/2}\left(-\frac{x}{2} + 2\right)$$

$f'(x) = xe^{-x/2}\left(-\frac{x}{2}+2\right)$

$f'<0$ | $f'>0$ | $f'<0$

$-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

$f'(x) = \text{DNE}?$

N/A

$f'(x) = 0?$

$xe^{-x/2}\left(-\frac{x}{2}+2\right) = 0$

$\boxed{x = 0}$ $\quad -\frac{x}{2} + 2 = 0$

$\boxed{x = 4}$

Critical pts: $\boxed{c_1 = 0,\ c_2 = 4}$

$f'(-1) = (-)(+)(+) = (-)$

$f'(1) = (+)(+)(+) = (+)$

$f'(5) = (+)(+)(-) = (-)$

f is increasing on $(0, 4)$
f is decreasing on $(-\infty, 0) \cup (4, \infty)$

3.] For the following functions, locate the critical values of f, use the First Derivative Test to locate the local extrema, and identify the absolute maximum and minimum values of the function on the specified interval (if they exist).

a.) $f(x) = 2x^5 - 5x^4 - 10x^3 + 4$ on $[-2, 4]$.

$f'(x) = 10x^4 - 20x^3 - 30x^2$

$f'(x) = DNE?$

N/A

$f'(x) = 0?$

$10x^4 - 20x^3 - 30x^2 = 0$

$10x^2(x^2 - 2x - 3) = 0$

$10x^2(x-3)(x+1) = 0$

$x = -1, 0, 3$

Critical Pts: $C_1 = -1, C_2 = 0$
$C_3 = 3$

Closed Interval Method:

$f(-2) = 2(-2)^5 - 5(-2)^4 - 10(-2)^3 + 4 = -60$

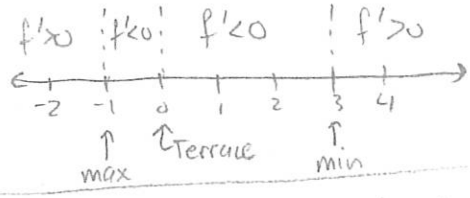
$f(-1) = 2(-1)^5 - 5(-1)^4 - 10(-1)^3 + 4 = 7$

$f(0) = 2(0)^5 - 5(0)^4 - 10(0)^3 + 4 = 4$

$\boxed{f(3)} = 2(3)^5 - 5(3)^4 - 10(3)^3 + 4 = -185$

$\boxed{f(4)} = 2(4)^5 - 5(4)^4 - 10(4)^3 + 4 = 132$

First derivative Test: $f'(x) = 10x^2(x-3)(x+1)$



Abs max at $X = 4$ Local min at $X = 3$
Abs min at $X = 3$ local max at $X = -1$

b.) $f(x) = x^{2/3}(5 - x)$ on $[-8, 8]$.

$f'(x) = \frac{2}{3}x^{-1/3}(5-x) + x^{2/3}(-1)$

$f'(x) = \frac{2}{3x^{1/3}}(5-x) - x^{2/3}$

$f'(x) = \frac{2(5-x) - 3x^{1/3}x^{2/3}}{3x^{1/3}}$

$f'(x) = \frac{10 - 2x - 3x}{3x^{1/3}}$

$f'(x) = \frac{5(2-x)}{3x^{1/3}}$

$f'(x) = DNE?$

$3x^{1/3} = 0$

$x = 0$

$f'(x) = 0?$

$5(2-x) = 0$

$x = 2$

Critical Pts: $C_1 = 0, C_2 = 2$

Closed Interval Method

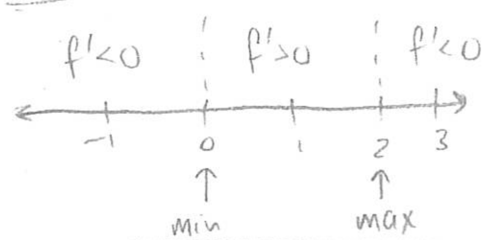
$\boxed{f(-8)} = (-8)^{2/3}(5-(-8)) = 4(13) = 52$

$f(0) = (0)^{2/3}(5-0) = 0(5) = 0$

$f(2) = (2)^{2/3}(5-2) = 3\sqrt[3]{4} \approx 4.76$

$\boxed{f(8)} = (8)^{2/3}(5-8) = 4(-3) = -12$

First Derivative Test: $f'(x) = \frac{5(2-x)}{3x^{1/3}}$



Abs min at $x = 8$ Local min at $x = 0$
Abs max at $x = -8$ Local max at $x = 2$