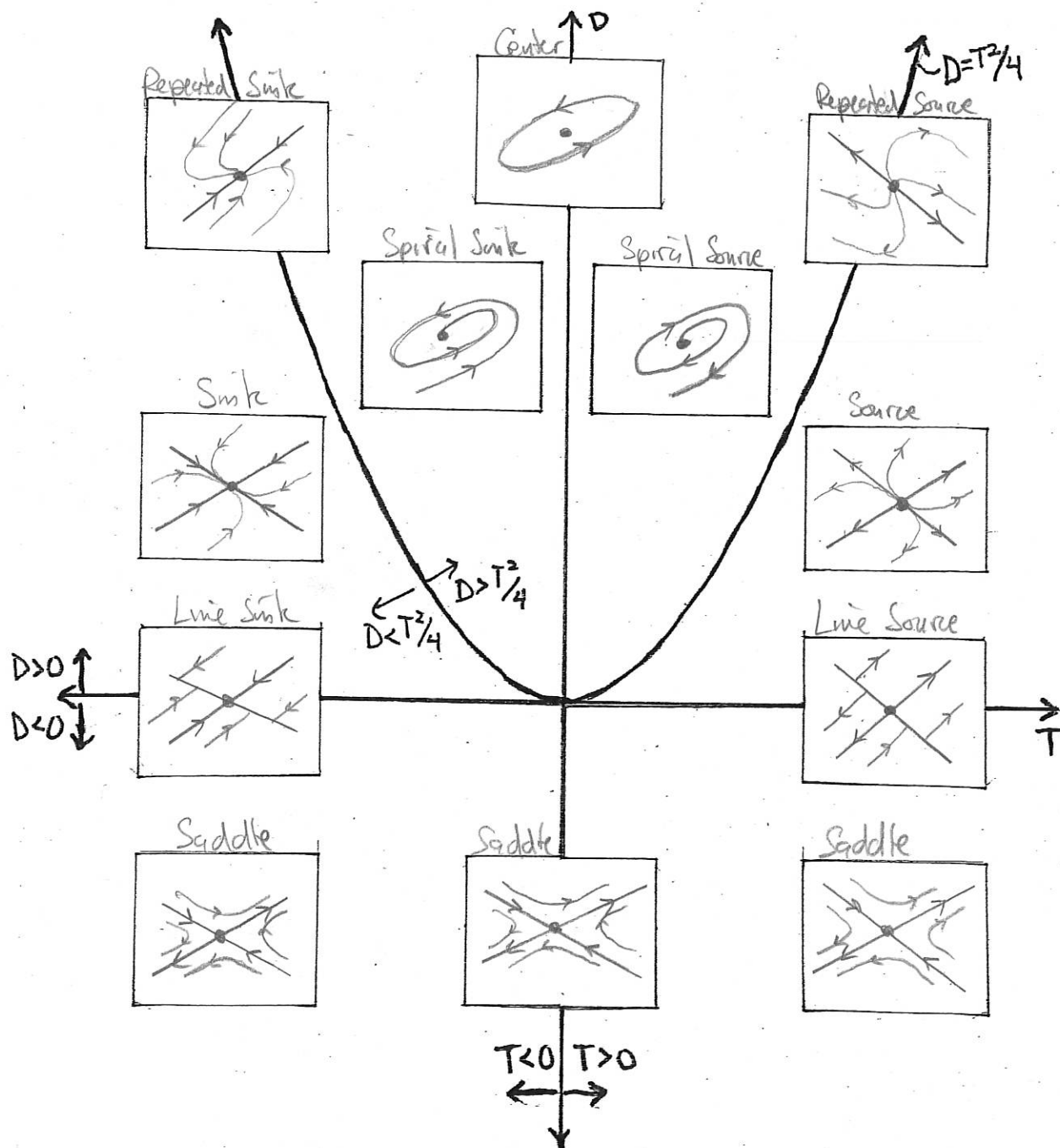


§3.7 (PART 1): TRACE-DETERMINANT PLANE

1.] Label the Trace-Determinant Plane below.



2.] For each system below, find the classify the stability of the origin using only the trace and determinant.

(a) $\frac{dx}{dt} = 2x + 2y$ $T = 8$ $D = 20$ $T^2/4 = 64/4 = 16$
 $\frac{dy}{dt} = -4x + 6y$ $T > 0$ $D > 0$ $D > T^2/4 \Rightarrow$ Spiral Source

(b) $\frac{dx}{dt} = -8x + 3y$ $T = -21$ $D = 98$ $T^2/4 = (21)^2/4 = 110.25$
 $\frac{dy}{dt} = 2x - 13y$ $T < 0$ $D > 0$ $D < T^2/4 \Rightarrow$ Sink

(c) $\frac{dx}{dt} = x + 4y$ $T = 0$ $D = 11$ $T^2/4 = 0$
 $\frac{dy}{dt} = -3x - y$ $T = 0$ $D > 0$ $D > T^2/4 \Rightarrow$ Center

(d) $\frac{dx}{dt} = 4x + 2y$ $T = 5$ $D = 0$ $T^2/4 = 25/4 = 6.25$
 $\frac{dy}{dt} = 2x + y$ $T > 0$ $D = 0$ $D < 6.25 \Rightarrow$ Line Source

(e) $\frac{dx}{dt} = -\frac{x}{2}$ $T = -1$ $D = \frac{1}{4}$ $T^2/4 = 1/4 = \frac{1}{4}$
 $\frac{dy}{dt} = x - \frac{y}{2}$ $T < 0$ $D > 0$ $D = T^2/4 \Rightarrow$ Repeated Sink

(f) $\frac{dx}{dt} = 2x + y$ $T = 6$ $D = 9$ $T^2/4 = 36/4 = 9$
 $\frac{dy}{dt} = -x + 4y$ $T > 0$ $D > 0$ $D = T^2/4 \Rightarrow$ Repeated Source

(g) $\frac{dx}{dt} = -3x - 5y$ $T = -2$ $D = 12$ $T^2/4 = 4/4 = 1$
 $\frac{dy}{dt} = 3x + y$ $T < 0$ $D > 0$ $D > T^2/4 \Rightarrow$ Spiral Sink

(h) $\frac{dx}{dt} = -3x + 4y$ $T = -8$ $D = -9$
 $\frac{dy}{dt} = 6x - 5y$ $T < 0$ $D < 0 \Rightarrow$ Saddle

If $D < 0 \Rightarrow$ Automatic Saddle!