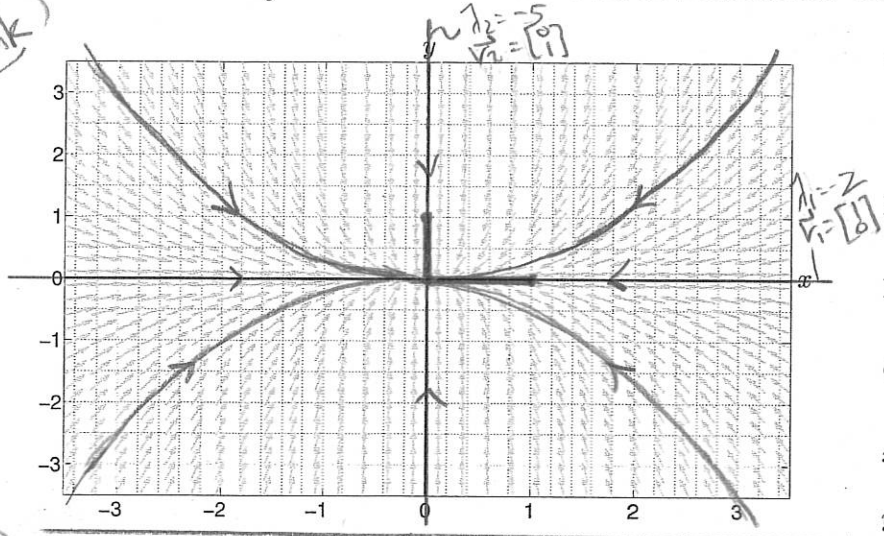
§3.3: Stability - Real Distinct Eigenvalues

Sink



Consider the system:

$$\frac{dy}{dt} = \begin{bmatrix} -2 & 0 \\ 0 & -5 \end{bmatrix} y$$

Eigenvalues: $\lambda_1 = -2, \quad \lambda_2 = -5$

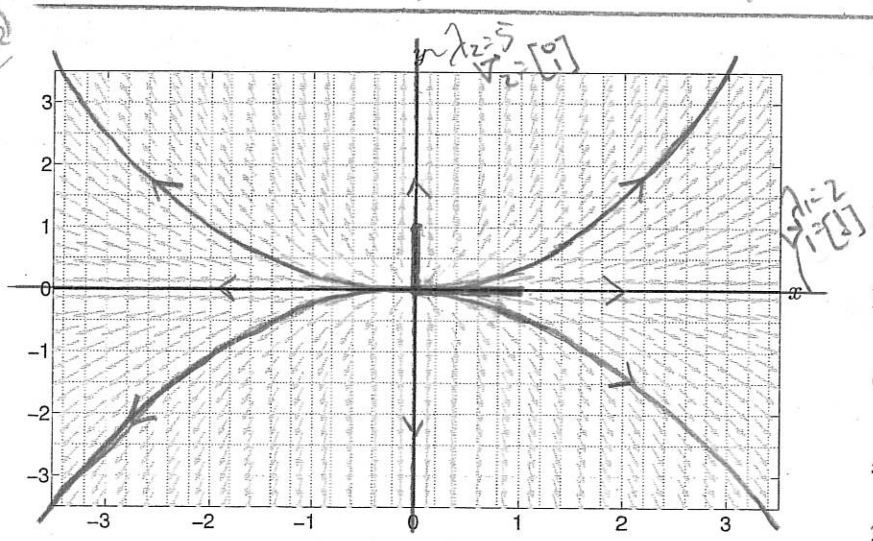
Eigenvectors: $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

General Solution: $\vec{y}(t) = C_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{-2t} + C_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t}$

$x(t) = C_1 e^{-2t}$

$y(t) = C_2 e^{-5t}$

Source



Consider the system:

$$\frac{dy}{dt} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} y$$

Eigenvalues: $\lambda_1 = 2, \quad \lambda_2 = 5$

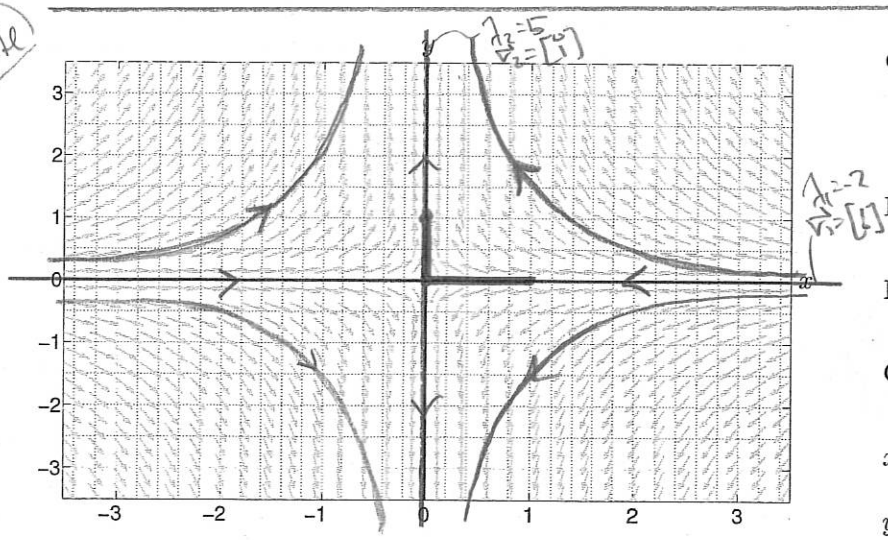
Eigenvectors: $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

General Solution: $\vec{y}(t) = C_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{2t} + C_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

$x(t) = C_2 e^{2t}$

$y(t) = C_2 e^{5t}$

Saddle



Consider the system:

$$\frac{dy}{dt} = \begin{bmatrix} -2 & 0 \\ 0 & 5 \end{bmatrix} y$$

Eigenvalues: $\lambda_1 = -2, \quad \lambda_2 = 5$

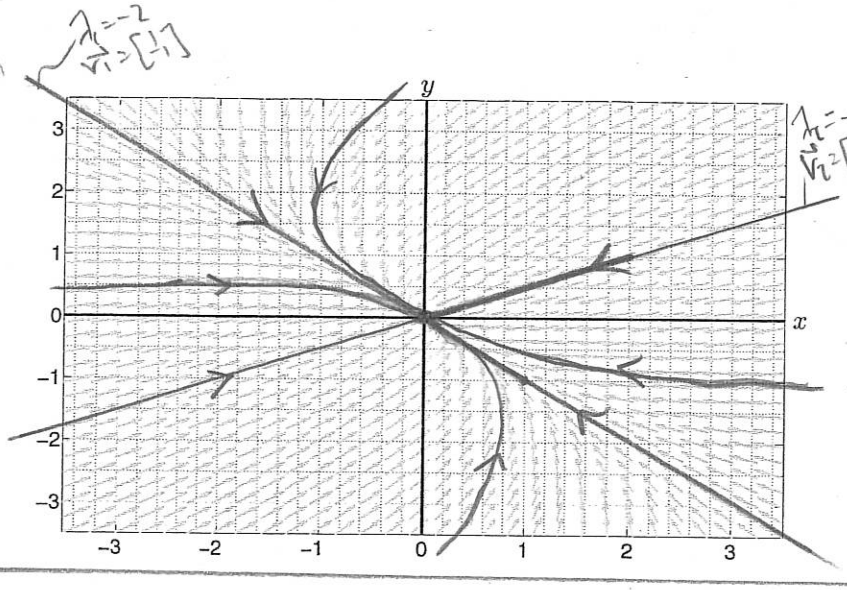
Eigenvectors: $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

General Solution: $\vec{y}(t) = C_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{-2t} + C_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

$x(t) = C_1 e^{-2t}$

$y(t) = C_2 e^{5t}$

Sink



Consider the system:

$$\frac{d\boldsymbol{y}}{dt}=\begin{bmatrix}-4 & -2\\ -1 & -3\end{bmatrix}\boldsymbol{y}$$

Eigenvalues: $\lambda_1=-2,\ \lambda_2=-5$

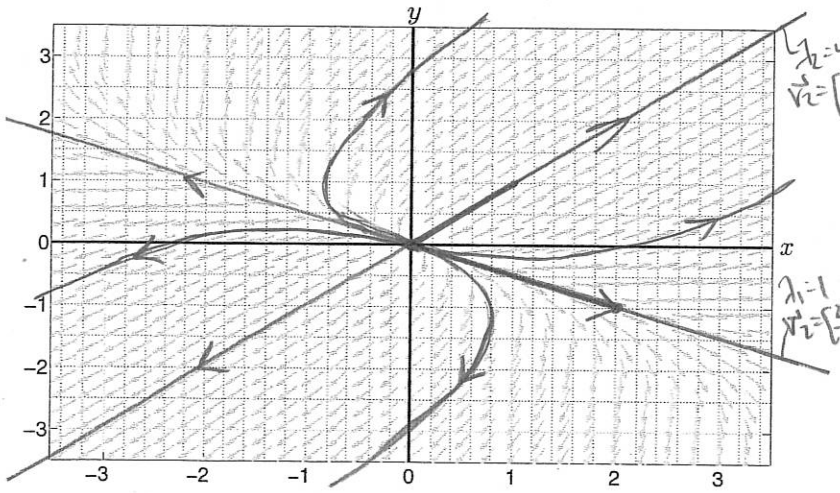
Eigenvectors: $\vec{v}_1=\begin{bmatrix}1\\-1\end{bmatrix},\ \vec{v}_2=\begin{bmatrix}2\\1\end{bmatrix}$

General Solution: $\vec{y}(t)=C_1\begin{bmatrix}1\\-1\end{bmatrix}e^{-2t}+C_2\begin{bmatrix}2\\1\end{bmatrix}e^{-5t}$

$x(t)=C_1e^{-2t}+2C_2e^{-5t}$

$y(t)=-C_1e^{-2t}+C_2e^{-5t}$

Source



Consider the system:

$$\frac{d\boldsymbol{y}}{dt}=\begin{bmatrix}2 & 2\\ 1 & 3\end{bmatrix}\boldsymbol{y}$$

Eigenvalues: $\lambda_1=1,\ \lambda_2=4$

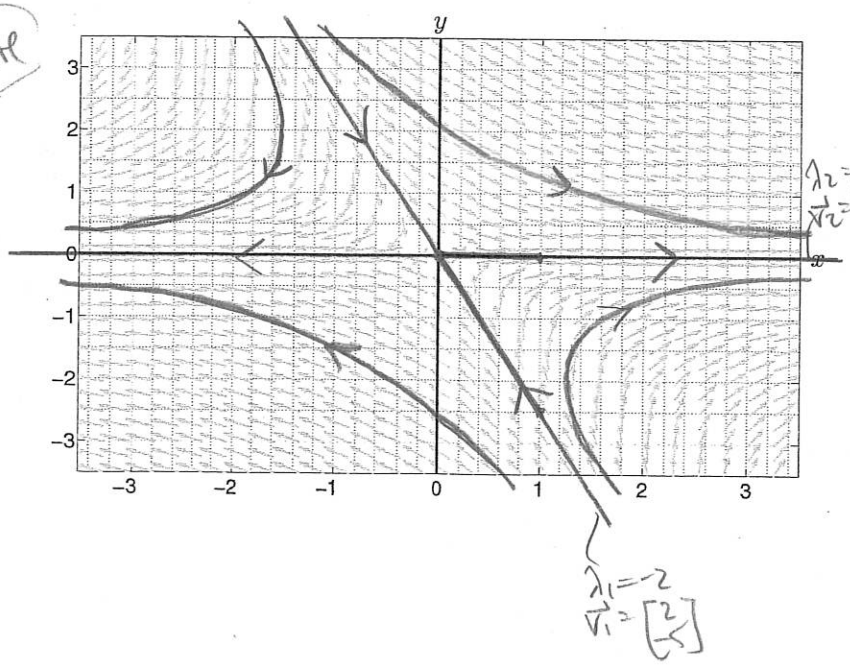
Eigenvectors: $\vec{v}_1=\begin{bmatrix}2\\-1\end{bmatrix},\ \vec{v}_2=\begin{bmatrix}1\\1\end{bmatrix}$

General Solution: $\vec{y}(t)=C_1\begin{bmatrix}2\\-1\end{bmatrix}e^{t}+C_2\begin{bmatrix}1\\1\end{bmatrix}e^{4t}$

$x(t)=2C_1e^{t}+C_2e^{4t}$

$y(t)=-C_1e^{t}+C_2e^{4t}$

Saddle



Consider the system:

$$\frac{d\boldsymbol{y}}{dt}=\begin{bmatrix}3 & 2\\ 0 & -2\end{bmatrix}\boldsymbol{y}$$

Eigenvalues: $\lambda_1=-2,\ \lambda_2=3$

Eigenvectors: $\vec{v}_1=\begin{bmatrix}2\\-5\end{bmatrix},\ \vec{v}_2=\begin{bmatrix}1\\0\end{bmatrix}$

General Solution: $\vec{y}(t)=C_1\begin{bmatrix}2\\-5\end{bmatrix}e^{-2t}+C_2\begin{bmatrix}1\\0\end{bmatrix}e^{3t}$

$x(t)=2C_1e^{-2t}+C_2e^{3t}$

$y(t)=-5C_1e^{-2t}$

Back of Section 5.3 worksheet:

$\frac{d\vec{y}}{dt} = \begin{bmatrix} -4 & -2 \\ -1 & -3 \end{bmatrix} \vec{y}$

$tr(A) = -7$

$\det(A) = 10$

E-values: $\lambda^2 + 7\lambda + 10 = 0$

$(\lambda+2)(\lambda+5) = 0$

$\boxed{\lambda_1 = -2, \ \lambda_2 = -5}$

E-vectors: $\boxed{\lambda_1 = -2:}$ $\begin{cases} (-4-(-2))x - 2y = 0 \\ -x + (-3-(-2))y = 0 \end{cases} \Rightarrow \begin{cases} -2x - 2y = 0 \\ -x - y = 0 \end{cases} \Rightarrow \begin{cases} y = -x \\ y = -x \end{cases}$

$\boxed{\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$

$\boxed{\lambda_2 = -5:}$ $\begin{cases} (-4-(-5))x - 2y = 0 \\ -x + (-3-(-5))y = 0 \end{cases} \Rightarrow \begin{cases} x - 2y = 0 \\ -x + 2y = 0 \end{cases} \Rightarrow \begin{cases} y = \frac{1}{2}x \\ y = \frac{1}{2}x \end{cases}$

$\boxed{\vec{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}}$

General Solution: $\vec{y}(t) = c_1 \vec{v}_1 e^{\lambda_1 t} + c_2 \vec{v}_2 e^{\lambda_2 t}$

$\Rightarrow \vec{y}(t) = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-5t}$

$\Rightarrow x(t) = c_1 e^{-2t} + 2c_2 e^{-5t}$

$y(t) = -c_1 e^{-2t} + c_2 e^{-5t}$

$\frac{d\vec{y}}{dt} = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} \vec{y}$

$tr(A) = 5$

$\det(A) = 4$

E-values: $\lambda^2 - 5\lambda + 4 = 0$

$(\lambda-4)(\lambda-1) = 0$

$\boxed{\lambda_1 = 1, \ \lambda_2 = 4}$

E-vectors: $\boxed{\lambda_1 = 1}$: $\begin{cases} (2-1)x + 2y = 0 \\ x + (3-1)y = 0 \end{cases} \Rightarrow \begin{cases} x + 2y = 0 \\ x + 2y = 0 \end{cases} \Rightarrow \begin{cases} y = -\frac{1}{2}x \\ y = -\frac{1}{2}x \end{cases}$

$\boxed{\vec{v}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}}$

$\boxed{\lambda_2 = 4}$: $\begin{cases} (2-4)x + 2y = 0 \\ x + (3-4)y = 0 \end{cases} \Rightarrow \begin{cases} -2x + 2y = 0 \\ x - y = 0 \end{cases} \Rightarrow \begin{cases} y = x \\ y = x \end{cases}$

$\boxed{v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

General Solution: $\vec{y}(t) = c_1 \vec{v}_1 e^{\lambda_1 t} + c_2 \vec{v}_2 e^{\lambda_2 t}$

$\Rightarrow \vec{y}(t) = c_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t}$

$\Rightarrow x(t) = 2c_1 e^{t} + c_2 e^{4t}$

$y(t) = -c_1 e^{t} + c_2 e^{4t}$

$\frac{d\vec{y}}{dt} = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix} \vec{y}$

$tr(A) = 1$

$det(A) = -6$

Evalues: $\lambda^2 - \lambda - 6 = 0$

$(\lambda - 3)(\lambda + 2) = 0$

$\boxed{\lambda_1 = -2, \lambda_2 = 3}$

E-vectors: $\boxed{\lambda_1 = -2}$: $\begin{cases} (3-(-2))x + 2y = 0 \\ 0x + (-2-(-2))y = 0 \end{cases} \Rightarrow \begin{cases} 5x + 2y = 0 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} y = -\frac{5}{2}x \end{cases}$

$\boxed{\vec{v}_1 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}}$

$\boxed{\lambda_2 = 3}$: $\begin{cases} (3-3)x + 2y = 0 \\ 0x + (-2-3)y = 0 \end{cases} \Rightarrow \begin{cases} 0x + 2y = 0 \\ 0x - 5y = 0 \end{cases} \Rightarrow \begin{cases} y = 0 \\ y = 0 \end{cases}$

$\boxed{\vec{v}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$

General Solution: $\vec{y}(t) = c_1 \vec{v}_1 e^{\lambda_1 t} + c_2 \vec{v}_2 e^{\lambda_2 t}$

$$\Rightarrow \vec{y}(t) = c_1 \begin{bmatrix} 2 \\ -5 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{5t}$$

$$\Rightarrow x(t) = 2c_1 e^{-2t} + c_2 e^{5t}$$

$$y(t) = -5c_1 e^{-2t}$$