§3.1: Linear Systems

1.] Let $A = \begin{bmatrix} 5 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & \pi \\ e^5 & 6 \end{bmatrix}$, $\boldsymbol{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, and $\boldsymbol{y} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$. Compute $A\boldsymbol{x}$, AB, $A(\boldsymbol{x}+\boldsymbol{y})$, and $\boldsymbol{x}\boldsymbol{y}$.

2.] Consider the following system:

$$\begin{aligned} \frac{dx}{dt} &= 5z \\ \frac{dy}{dt} &= -5x + y \\ \frac{dz}{dt} &= -y + 6z \end{aligned}$$

Define $\boldsymbol{y}$ and A so that this system is represented as $\dfrac{d\boldsymbol{y}}{dt} = A\boldsymbol{y}$.

3.] Consider the system

$$\frac{d\boldsymbol{y}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix} \boldsymbol{y}.$$

Show that both of the solutions below satisfy this equation. Then show that $\boldsymbol{y}(t) = -\boldsymbol{y_1}(t) + 3\boldsymbol{y_2}(t)$ also satisfies this equation.

$$\boldsymbol{y_1}(t) = \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix}, \qquad \boldsymbol{y_2}(t) = \begin{bmatrix} -e^{-4t} \\ 2e^{-4t} \end{bmatrix}.$$