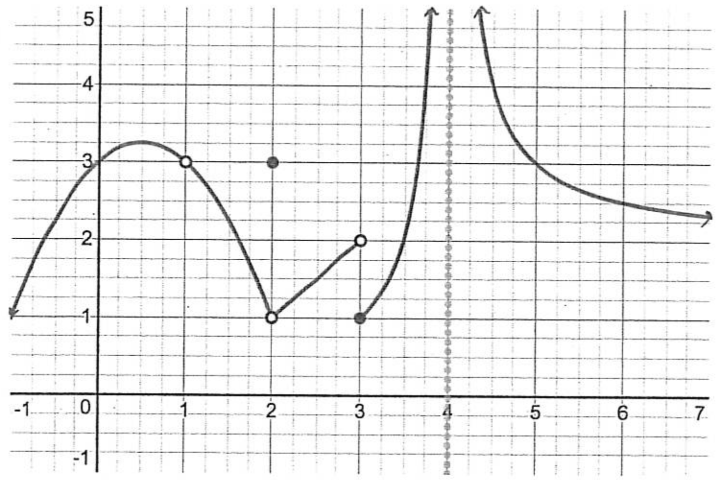
§2.6: CONTINUITY

1.] Consider the graph of a piecewise function $f(x)$ below:



Domain:
$(-\infty,1)\cup(1,4)\cup(4,\infty)$

a.) Compute the following:

Not Continuous *i.* $\lim_{x\to 1^-} f(x) = 3$ *ii.* $\lim_{x\to 1^+} f(x) = 3$ *iii.* $\lim_{x\to 1} f(x) = 3$ *iv.* $f(1) =$ DNE

b.) Compute the following:

Not Continuous *i.* $\lim_{x\to 2^-} f(x) = 1$ *ii.* $\lim_{x\to 2^+} f(x) = 1$ *iii.* $\lim_{x\to 2} f(x) = 1$ *iv.* $f(2) = 3$

c.) Compute the following:

Not Continuous *i.* $\lim_{x\to 3^-} f(x) = 2$ *ii.* $\lim_{x\to 3^+} f(x) = 1$ *iii.* $\lim_{x\to 3} f(x) =$ DNE *iv.* $f(3) = 1$

d.) Compute the following:

Not Continuous *i.* $\lim_{x\to 4^-} f(x) = \infty$ *ii.* $\lim_{x\to 4^+} f(x) = \infty$ *iii.* $\lim_{x\to 4} f(x) = \infty$ *iv.* $f(4) =$ DNE

e.) Compute the following:

Continuous *i.* $\lim_{x\to 6^-} f(x) = 2.5$ *ii.* $\lim_{x\to 6^+} f(x) = 2.5$ *iii.* $\lim_{x\to 6} f(x) = 2.5$ *iv.* $f(6) = 2.5$

f.) For what values of x is this function discontinuous? For each point of discontinuity, briefly explain *why* it is not continuous.

The function is discontinuous at $x = 1, 2, 3, 4$.

- $x=1$: function value doesn't exist. (1 is not in domain)
- $x=2$: the limit is different from the function value.
- $x=3$: the limit doesn't exist.
- $x=4$: the function value doesn't exist. (4 is not in domain)

2.] Determine if the following functions are continuous at $x = a$:

a.) $f(x) = \dfrac{3x^2+2x+1}{x-1}$, $a = 1$.

$f(1) = \frac{3+2+1}{1-1} = \frac{6}{0} = \text{DNE}$

Not Continuous!

(1 is not in domain)

b.) $f(x) = \begin{cases} \frac{x^2-4x+3}{x-3} & \text{if } x \neq 3 \\ 2 & \text{if } x = 3 \end{cases}$, $a = 3$.

$f(3) = 2$ ⟶ SAME

$\lim_{x\to 3} \frac{\cancel{(x-3)}(x-1)}{\cancel{x-3}} = \lim_{x\to 3}(x-1) = 3-1 = 2$

Continuous!

c.) $f(x) = \dfrac{2x^2+3x+1}{x^2+5x}$, $a = 5$.

$f(5) = \frac{2(5)^2+3(5)+1}{5^2+5(5)} = \frac{50+15+1}{50} = \frac{66}{50}$

$\lim_{x\to 5} f(x) = f(5) = \frac{66}{50}$ — Same!

Continuous!

d.) $f(x) = \dfrac{e^{2x}-1}{e^x-1}$, $a = 0$.

$f(0) = \frac{e^{2(0)}-1}{e^0-1} = \frac{e^0-1}{e^0-1} = \frac{1-1}{1-1} = \frac{0}{0} = \text{DNE}$

$\lim_{x\to 0} \frac{e^{2x}-1}{e^x-1} = \lim_{x\to 0} \frac{(e^x-1)(e^x+1)}{e^x-1} = e^0+1 = 2$

Not Continuous!

3.] Determine the interval(s) on which the following function is continuous. Be sure to consider left- and right-continuity.

$$f(x) = \begin{cases} 2x & \text{if } x < 1 \\ x^2+3x & \text{if } x \geq 1 \end{cases}$$

$f(1) = 1^2 + 3(1) = 4$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2x = 2(1) = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2+3x = 1^2+3(1) = 4$

- f is right continuous at $x=1$, but overall it is discontinuous at $x=1$.
- f is continuous on $(-\infty,1)$ and $[1,\infty)$.

4.] Use the Intermediate Value Theorem to show that the following equation has a solution on the given interval:

$$\underbrace{2x^3+x}_{f(x)} = 2, \qquad (-1,1).$$

Let $f(x) = 2x^3+x$, then we know $f(x)$ is continuous everywhere.

Also, we have

$f(-1) = 2(-1)^3+(-1) = -2-1 = -3 < 2$

$f(1) = 2(1)^3+1 = 2+1 = 3 > 2$.

Therefore, $f(x) = 2$ somewhere in the interval $(-1,1)$.