§2.3: Techniques for Computing Limits

1.] Determine the following limits:

a.) $\lim_{x \to 3}(2x-4)$

$= 2(3)-4$

$= \boxed{2}$

b.) $\lim_{x \to 6} \pi$

$= \boxed{\pi}$

2.] Assuming $\lim_{x\to 1} f(x) = 8$, $\lim_{x\to 1} g(x) = 3$, and $\lim_{x\to 1} h(x) = 2$, compute the following limits:

a.) $\lim_{x\to 1}(f(x) - g(x)) = \lim_{x\to 1} f(x) - \lim_{x\to 1} g(x) = 8-3 = \boxed{5}$

b.) $\lim_{x\to 1}\left[\frac{f(x)}{g(x)-h(x)}\right] = \frac{\lim_{x\to 1} f(x)}{\lim_{x\to 1} g(x) - \lim_{x\to 1} h(x)} = \frac{8}{3-2} = \frac{8}{1} = \boxed{8}$

c.) $\lim_{x\to 1} \sqrt[3]{f(x)g(x)+3}$

$= \lim_{x\to 1}(f(x)g(x)+3)^{1/3} = \left[\left(\lim_{x\to 1} f(x)\right)\left(\lim_{x\to 1} g(x)\right)+3\right]^{1/3} = ((8)(3)+3)^{1/3} = (27)^{1/3} = \boxed{3}$

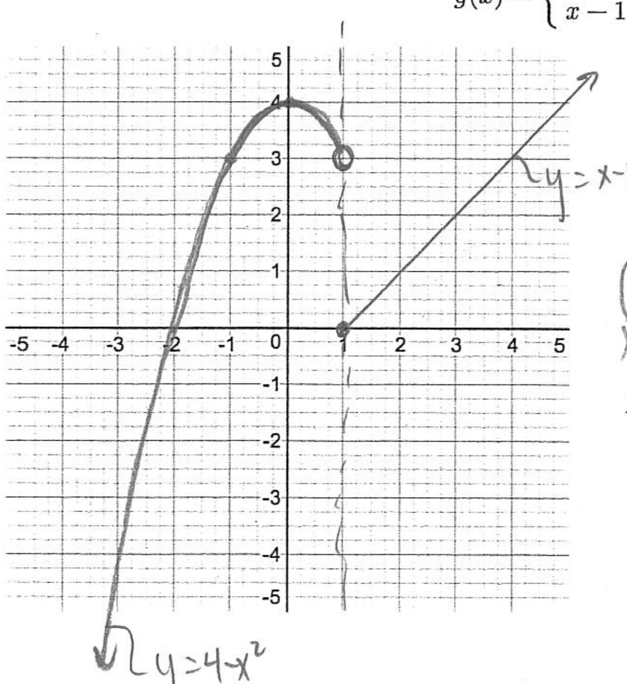
3.] Compute the following limit: $\lim_{x\to 0} \frac{3}{\sqrt{16+3x}+4}$

Plug it in!
0 is in domain!

$= \frac{3}{\sqrt{16+3(0)}+4} = \frac{3}{\sqrt{16}+4} = \frac{3}{4+4} = \boxed{\frac{3}{8}}$

4.] Consider the piecewise function $g(x)$ given below. Sketch the graph on the grid paper and determine $\lim_{x\to 1} g(x)$.

$$g(x) = \begin{cases} 4-x^2 & \text{if } x < 1 \\ x-1 & \text{if } x \geq 1 \end{cases}$$



$\lim_{x\to 1} g(x) = \boxed{\text{DNE}}$

$\lim_{x\to 1^-}(4-x^2)$

$= 4-(1)^2$

$= 4-1$

$= 3$

$\lim_{x\to 1^+}(x-1)$

$= 1-1$

$= 0$

Not equal!

5.] Determine the following limits and provide a sketch of the graph of the function.

a.) $\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$

$= \lim_{x\to 1} (x+1)$

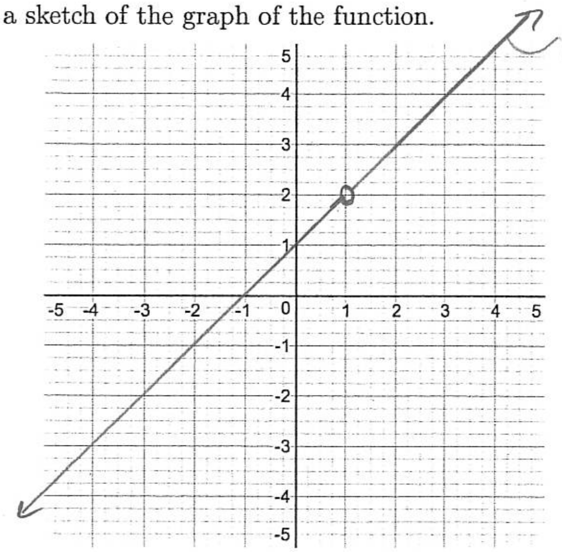
$= 1+1$

$= \boxed{2}$

Hole at $(1,2)$

The limit exists but the function value does not exist ($f(1)=$ DNE)

$y = \frac{x^2-1}{x-1}$



Domain:

$(-\infty,1)\cup(1,\infty)$

b.) $\lim_{x\to 9} \frac{9-x}{3-\sqrt{x}} = \lim_{x\to 9} \frac{\cancel{(3-\sqrt{x})}(3+\sqrt{x})}{\cancel{3-\sqrt{x}}}$

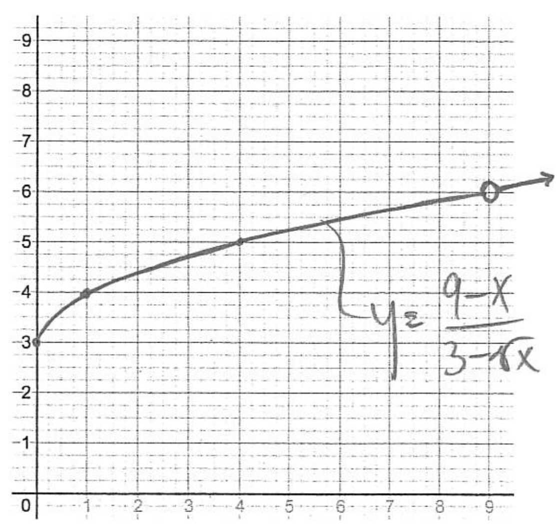
$= \lim_{x\to 9} 3+\sqrt{x}$

$= 3+\sqrt{9}$

$= \boxed{6}$

Hole at $(9,6)$

The limit exists but the function value does not exist ($f(9)=$ DNE)



Domain:

$[0,9)\cup(9,\infty)$

6.] Evaluate the following limit and describe the graph of the function:

$$\lim_{x\to 2} \frac{x^2-6x+8}{x^2-4}.$$

$\lim_{x\to 2} \frac{(x-4)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \lim_{x\to 2} \frac{x-4}{x+2} = \frac{2-4}{2+2} = \frac{-2}{4} = \boxed{-\frac{1}{2}}$

The graph of this function looks like $y = \frac{x-4}{x+2}$, but with a hole at the point $(2,-\frac{1}{2})$.