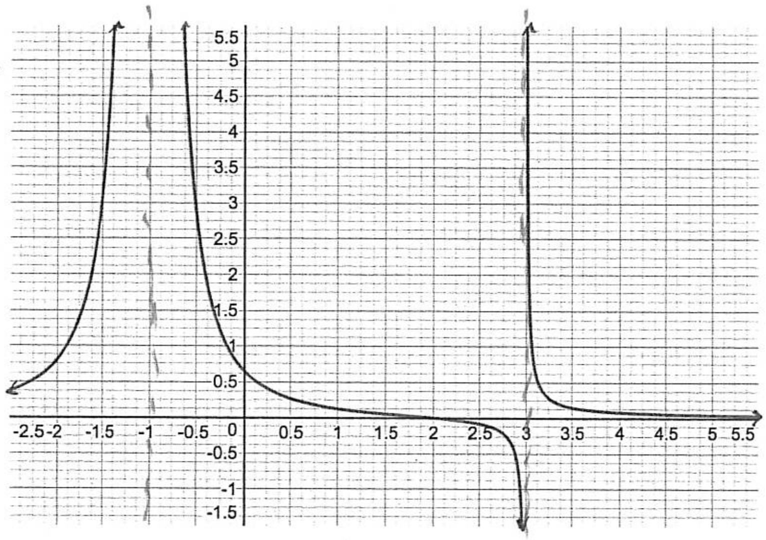
§2.4: Infinite Limits

1.] The graph of the rational functions

$$f(x) = \frac{x-2}{(x+1)^2(x-3)}$$

is given below. Determine the domain of this function and evaluate the following limits.

Domain: $x \neq -1, 3$
$(-\infty,-1)\cup(-1,3)\cup(3,\infty)$



a.) $\lim_{x\to -1^-} f(x) = \infty$

b.) $\lim_{x\to -1^+} f(x) = \infty$

c.) $\lim_{x\to -1} f(x) = \infty$

d.) $f(-1) = \text{DNE}$

e.) $\lim_{x\to 2^-} f(x) = 0$

f.) $\lim_{x\to 2^+} f(x) = 0$

g.) $\lim_{x\to 2} f(x) = 0$

h.) $f(2) = 0$

i.) $\lim_{x\to 3^-} f(x) = -\infty$

j.) $\lim_{x\to 3^+} f(x) = \infty$

k.) $\lim_{x\to 3} f(x) = \text{DNE}$

l.) $f(3) = \text{DNE}$

2.] Consider the following rational function:

$$f(x) = \frac{x^2-3x+2}{x-3} = \frac{(x-1)(x-2)}{x-3}$$

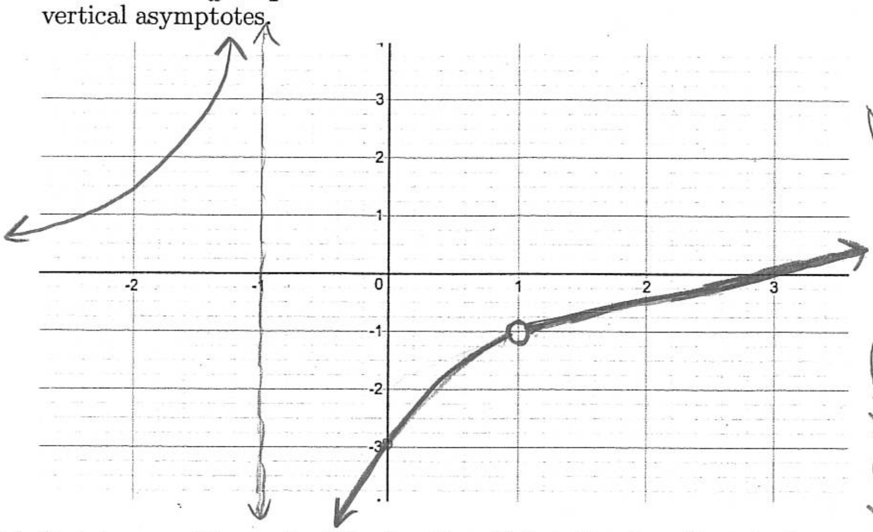
Determine the domain of this function and evaluate the following limits:

(a) $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{(x-1)(x-2)}{x-3} = \frac{(2)(1)}{0^-} = \boxed{-\infty}$

(b) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{(x-1)(x-2)}{x-3} = \frac{(2)(1)}{0^+} = \boxed{\infty}$

(c) $\lim_{x\to 3} f(x) = \boxed{\text{DNE}}$ since the limits above are not equal!

3.] Let $f(x) = \dfrac{x^2 - 4x + 3}{x^2 - 1}$. Determine the domain of $f(x)$ and sketch a graph of $f(x)$ below, indicating all vertical asymptotes.



$f(x) = \dfrac{(x-1)(x-3)}{(x-1)(x+1)}$

Domain: $x \neq -1, 1$

$(-\infty,-1) \cup (-1,1) \cup (1,\infty)$

$\lim_{x \to 1} f(x) = \dfrac{1-3}{1+1} = \dfrac{-2}{2} = -1$ Hole

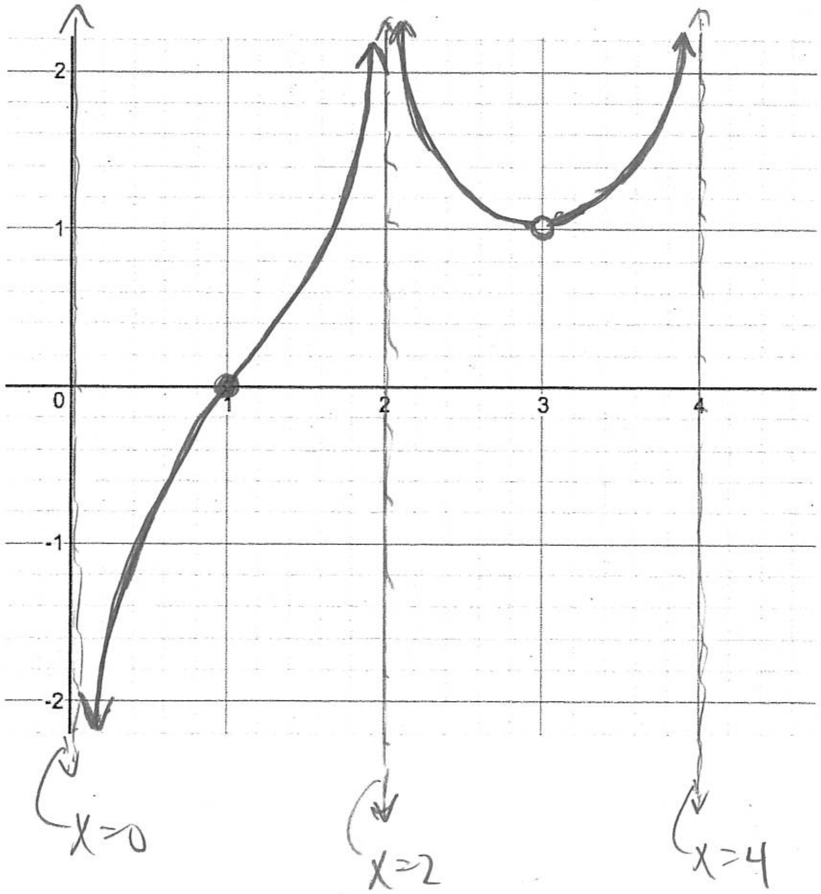
$\lim_{x \to -1^-} f(x) = \dfrac{(-2)(-4)}{(-2)(0^-)} = \infty$

$\lim_{x \to -1^+} f(x) = \dfrac{(-2)(-4)}{(-2)(0^+)} = -\infty$

$\Big\}$ VA

4.] Sketch a possible graph of the function $f(x)$, indicating all vertical asymptotes, that satisfies the following conditions on the interval $[0, 4]$:

$f(1) = 0,\quad f(3) = \text{DNE},\quad \lim_{x\to 3} f(x) = 1,\quad \lim_{x\to 0^+} f(x) = -\infty,\quad \lim_{x\to 2} f(x) = \infty,\quad \lim_{x\to 4^-} f(x) = \infty$



- $f(1) = 0$: point at $(1,0)$
- $f(3) = \text{DNE}$
- $\lim_{x\to 3} f(x) = 1$ $\Big\}$ Hole at $(3,1)$
- $\lim_{x\to 0^+} f(x) = -\infty$: VA at $x=0$
- $\lim_{x\to 2} f(x) = \infty$: VA at $x=2$
- $\lim_{x\to 4^-} f(x) = \infty$: VA at $x=4$