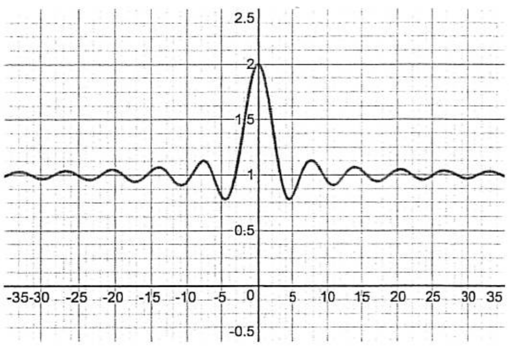
§2.5: Limits at Infinity

Answer: Yes

1.] Evaluate $\lim_{x\to\infty} 1+\dfrac{\sin(x)}{x}$. Does this function have a horizontal asymptote?



$$\lim_{x\to\infty}\left(1+\frac{\sin(x)}{x}\right) = \lim_{x\to\infty} 1 + \frac{\lim_{x\to\infty}\sin(x)}{\lim_{x\to\infty} x} = 1 + \frac{(\text{something in } [-1,1])}{\infty}$$
$$= 1+0 = \boxed{1}$$

HA at $y=1$

$$\lim_{x\to-\infty}\left(1+\frac{\sin(x)}{x}\right) = \lim_{x\to-\infty} 1 + \frac{\lim_{x\to-\infty}\sin(x)}{\lim_{x\to-\infty} x} = 1+0 = \boxed{1}$$

2.] Determine the limits at infinity for the following polynomial: $f(x) = 4x^4 - 2x^3 + x - 100$

$$\lim_{x\to\infty}(4x^4-2x^3+x-100) = \lim_{x\to\infty} x^4(4-2/x+1/x^3-100/x^4) = (\infty)(4) = \boxed{\infty}$$
$$\lim_{x\to-\infty}(4x^3-2x^3+x-100) = \lim_{x\to-\infty} x^4(4-2/x+1/x^3-100/x^4) = (\infty)(4) = \boxed{\infty}$$

3.] Determine the end behavior of the following rational functions:

a.) $f(x) = \dfrac{2x+1}{x^2-1}$ ← HA at $y=0$

$$\lim_{x\to\infty}\frac{2x+1}{x^2-1} = \lim_{x\to\infty}\frac{x(2+1/x)}{x^2(1-1/x^2)} = \lim_{x\to\infty}\frac{2+\frac{1}{x}}{x(1-1/x^2)} = \frac{2}{(\infty)(1)} = \boxed{0}$$
$$\lim_{x\to-\infty}\frac{2x+1}{x^2-1} = \lim_{x\to-\infty}\frac{x(2+1/x)}{x^2(1-1/x^2)} = \lim_{x\to-\infty}\frac{2+1/x}{x(1-1/x^2)} = \frac{2}{(-\infty)(1)} = \boxed{0}$$

b.) $g(x) = \dfrac{20x^4-6x^2-10}{4x^4+8x^2+1}$ ← HA at $y=5$.

$$\lim_{x\to\infty}\frac{20x^4-6x^2-10}{4x^4+8x^2+1} = \lim_{x\to\infty}\frac{x^4(20-6/x^2-10/x^4)}{x^4(4+8/x^2+1/x^4)} = \lim_{x\to\infty}\frac{20-6/x^2-10/x^4}{4+8/x^2+1/x^4} = \frac{20}{4} = \boxed{5}$$
$$\lim_{x\to-\infty}\frac{20x^4-6x^2-10}{4x^4+8x^2+1} = \lim_{x\to-\infty}\frac{x^4(20-6/x^2-10/x^4)}{x^4(4+8/x^2+1/x^4)} = \lim_{x\to\infty}\frac{20-6/x^2-10/x^4}{4+8/x^2+1/x^4} = \frac{20}{4} = \boxed{5}$$

c.) $h(x) = \dfrac{-x^3+x-5}{3x+1}$

$$\lim_{x\to\infty}\frac{-x^3+x-5}{3x+1} = \lim_{x\to\infty}\frac{x^3(-1+1/x-5/x^3)}{x(3+1/x^3)} = \lim_{x\to\infty}\frac{x^2(-1+1/x-5/x^3)}{3+1/x^2} = \frac{(\infty)(-1)}{3} = \boxed{-\infty}$$
$$\lim_{x\to-\infty}\frac{-x^3+x-5}{3x+1} = \lim_{x\to-\infty}\frac{x^3(-1+1/x-5/x^3)}{x(3+1/x^3)} = \lim_{x\to-\infty}\frac{x^2(-1+1/x-5/x^3)}{3+1/x^3} = \frac{(\infty)(-1)}{3} = \boxed{-\infty}$$

4.] Determine the end behavior of the following algebraic function:

$$f(x) = \frac{4x^3+1}{\sqrt{16x^6+x^2-1}}$$

$$\lim_{x\to\infty} \frac{4x^3+1}{\sqrt{16x^6+x^2-1}} = \lim_{x\to\infty} \frac{x^3(4+1/x^3)}{\sqrt{x^6(16+1/x^4-1/x^6)}}$$

$$= \lim_{x\to\infty} \frac{x^3(4+1/x^3)}{|x^3|\sqrt{16+1/x^4-1/x^6}}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^3}(4+1/x^3)}{\cancel{x^3}\sqrt{16+1/x^4-1/x^6}}$$

$$= \frac{4+0}{\sqrt{16+0-0}}$$

$$= \frac{4}{\sqrt{16}}$$

$$= \boxed{1}$$

HA at $y=1$

$$\lim_{x\to-\infty} \frac{4x^3+1}{\sqrt{16x^6+x^2-1}} = \lim_{x\to-\infty} \frac{x^3(4+1/x^3)}{\sqrt{x^6(16+1/x^4-1/x^6)}}$$

$$= \lim_{x\to-\infty} \frac{x^3(4+1/x^3)}{|x^3|\sqrt{16+1/x^4-1/x^6}}$$

$$= \lim_{x\to-\infty} \frac{\cancel{x^3}(4+1/x^3)}{(-\cancel{x^3})\sqrt{16+1/x^4-1/x^6}}$$

$$= \frac{4+0}{(-1)\sqrt{16+0-0}}$$

$$= \frac{4}{-\sqrt{16}}$$

$$= \boxed{-1}$$

HA at $y=-1$

5.] Determine the end behavior of the following transcendental functions:

a.) $f(x) = -3e^{-x}$

$$\lim_{x\to\infty} -3e^{-x} = -3\left(\lim_{x\to\infty} e^{-x}\right) = (-3)(0) = \boxed{0}$$

$$\lim_{x\to-\infty} -3e^{-x} = -3\left(\lim_{x\to-\infty} e^{-x}\right) = -3\left(e^{\infty}\right) = -3(\infty) = \boxed{-\infty}$$

b.) $g(x) = |\ln(x)|$

$$\lim_{x\to\infty} |\ln(x)| = \left|\lim_{x\to\infty} \ln(x)\right| = |\infty| = \boxed{\infty}$$

$$\lim_{x\to-\infty} |\ln(x)| = \left|\lim_{x\to-\infty} \ln(x)\right| = \boxed{DNE}$$ (x can't be negative)

c.) $h(x) = \sin(x)$

$$\lim_{x\to\infty} \sin(x) = DNE$$

$$\lim_{x\to-\infty} \sin(x) = DNE$$

$\Big\}$ Sin(x) doesn't get arbitrarily close to any single value as $x \to \pm\infty$.